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Chapter 1

Number Systems

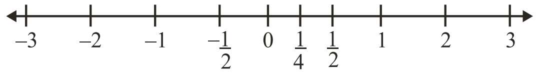
INTRODUCTION

We all know the numbers. We are playing with the numbers since our childhood. All the numbers which we studied till now are rational numbers. We also studied the representation of rational numbers on number line and about their basic algebraic operations like addition, subtraction, multiplication, division, L.C.M., H.C.F., etc of rational numbers.

In this chapter, we shall learn a new type of numbers called Irrational numbers. We will also learn the representation of irrational numbers on the number line and basic operations related to irrational numbers. All the rational and irrational numbers taken together are known as Real Numbers.

NUMBER LINE

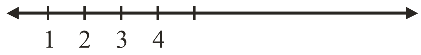
Representation of various types of numbers on the number line.



Various Types of Numbers

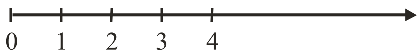
1. Set of Natural Numbers, N = {1, 2, 3, …}

representation of N on number line



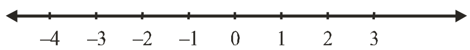
2. Set of whole numbers, W = {0, 1, 2, 3, …}

number line of W (whole numbers) :



3. Set of integers,
   Z = {…, –3, –2, –1, 0, 1, 2, 3, … }

number line of Z (integers) :



4. Rational numbers : A number 'r' is called a rational number, if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

   Rational numbers, $Q = \left\{ \frac{p}{q} : p, q \in I, q \neq 0 \right\}$

NOTE: The rational numbers also include the natural numbers, whole numbers and integers.

IRRATIONAL NUMBERS

A number 's' is called irrational, if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Examples are : $\sqrt{2}, \sqrt{3}, \sqrt{15}$, Π, 0.10110111011110...

NOTE: When we use the symbol $\sqrt{\ }$, we assume that it is the positive square root of the number. So $\sqrt{4}$ = 2, though both 2 and – 2 are square roots of 4.

REAL NUMBERS

The set of rational numbers and irrational numbers form a set of real numbers which is denoted by R.

Remark

Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.

NOTE: (i) All integers are rational numbers.

(ii) The square root of every perfect square number is rational. eg. $\sqrt{4} = 2, \sqrt{9} = 3, \sqrt{16} = 4$ etc. are all rational numbers.

(iii) The square root of any positive number which is not a perfect square is an irrational number.

E.g.: $\sqrt{5}, \sqrt{3}, \sqrt{10}, \sqrt{12}, \sqrt{3.4}, \sqrt{0.748}$ etc.

(iv) π is an irrational number, which is actually the ratio of circumference to the diameter of a circle i.e. $\pi = \frac{c}{d}$, where c and d are the circumference and diameter of a circle. Approximate value of π is taken as $\frac{22}{7}$ or 3.14

FINDING RATIONAL NUMBERS BETWEEN TWO NUMBERS

Method to find two or more rational numbers between two numbers p and q:

In general, there are infinitely many rational numbers between any two given rational numbers.

If $p < q$, then one of the number be $p < \dfrac{p+q}{2} < q$ and others will be in continuation as $p < \dfrac{p+(p+q)/2}{2} < \dfrac{p+q}{2}$

ILLUSTRATION : 1

Find six rational numbers between $\dfrac{4}{7}$ and $\dfrac{1}{7}$.

SOLUTION:

Denominator of both the given rational numbers $\dfrac{4}{7}$ and $\dfrac{1}{7}$ are equal. Now the difference of numerator = 4 – 1 = 3, which is not greater than 6 (i.e. the number of rational numbers is to be found out). On multiplying both numerator and denominator of $\dfrac{4}{7}$ and $\dfrac{1}{7}$ by 3, we get

$$\frac{4}{7} = \frac{4\times3}{7\times3} = \frac{12}{21}, \quad \frac{1}{7} = \frac{1\times3}{7\times3} = \frac{3}{21}$$

Now the difference of numerator of $\dfrac{12}{21}$ and $\dfrac{3}{21}$ = 12 – 3 = 9, which is greater than 6.

Also, $\dfrac{3}{21} < \dfrac{12}{21}$ therefore $\dfrac{4}{21}, \dfrac{5}{21}, \dfrac{6}{21}, \dfrac{7}{21}, \dfrac{8}{21}$ and $\dfrac{9}{21}$ are one set of six rational numbers between $\dfrac{4}{7}$ and $\dfrac{1}{7}$.

DECIMAL REPRESENTATION OF RATIONAL NUMBERS IN THE FORM $\frac{p}{q}$

Every rational number in the form $\dfrac{p}{q}$ can be expressed to its equivalent decimal form.

For example $\dfrac{1}{5} = 0.2$, $\quad \dfrac{1}{3} = 0.333..... = 0.\overline{3}$, etc

Generally, we use long division method to get the decimal form.

For example to get the decimal form of $\dfrac{13}{7}$, we simply divide 13 by 7 is shown below

$$\begin{array}{r|l|l}
7 & 13 & 1.857142 \\
 & \underline{7} & \\
 & 60 & \\
 & \underline{56} & \\
 & 40 & \\
 & \underline{35} & \\
 & 50 & \\
 & \underline{49} & \\
 & 10 & \\
 & \underline{7} & \\
 & 30 & \\
 & \underline{28} & \\
 & 20 & \\
 & \underline{14} & \\
 & 6 &
\end{array}$$

$\therefore \quad \dfrac{13}{7} = 1.857142857142.... = 1.\overline{857142}$

Terminating Decimal Expansions

In this case, the decimal expansion terminates or ends after a finite number of steps. We call such a decimal expansion as terminating.

Non-terminating Recurring Expansions

In this case we have a repeating block of digits in the quotient. We say that this expansion is non-terminating recurring.

CONVERSION OF RATIONAL NUMBER IN DECIMAL FORM TO ITS SIMPLEST $\frac{p}{q}$ FORM

Type-I : Conversion of a Terminating Decimal Number to its Simplest $\frac{p}{q}$ Form.

Step 1 : Obtain the rational number.
Step 2 : Determine the number of digits in its decimal part.
Step 3 : Remove decimal point from the given number and write 1 as its denominator followed by as many zeros as the total number of digits in the decimal part of the given number.
Step 4 : Write the number obtained in step-3 in its simplest form (i.e. the form in which there is no common factor other than 1 in its numerator and denominator).

The number so obtained is the required $\frac{p}{q}$ form.

ILLUSTRATION : 2

Convert rational number 2.348 in simplest $\frac{p}{q}$ form.

SOLUTION :

Given rational number = 2.348
There are three digits in the decimal part.

$$\therefore \quad 2.348 = \frac{2348}{1000}$$

Now, write $\frac{2348}{1000}$ in its lowest form.

$$2.348 = \frac{2348}{1000} = \frac{1174}{500} = \frac{587}{250}$$, which is the required form.

Type-II : Conversion of Non-terminating Repeating Decimal Number to its Simplest $\frac{p}{q}$ Form.

Step 1 : Put the given number equal to x and consider it as equation number say (i)
Step 2 : If there are some non-repeating digits after decimal on the right hand side of the equation (i), then count the number of non-repeating digits after the decimal point. Let it be n. Otherwise go to step 4 directly.
Step 3 : Multiply both sides of equation (i) by $(10)^n$, so that on the right hand side of the decimal point only the repeating digit(s) is/are left. Consider the equation so obtained as equation number (ii).
Step 4 : Multiply both sides of the equation (ii) by $(10)^m$, where m is the number of digit(s) which repeats on the right hand side after decimal point. Consider the equation so obtained as equation number (iii).
Step 5 : Subtract the equation (ii) from equation (iii).
Step 6 : Divide both sides of the equation obtained in step-5 by the coefficient of x.
Step 7 : Write the rational number on the right hand side of equation obtained in step-6 in the simplest $\frac{p}{q}$ form. This simplest $\frac{p}{q}$ form is the required form.

ILLUSTRATION : 3

Express 6.48261261261………. in the simplest $\frac{p}{q}$ form.

SOLUTION :

Let x = 6.48261264261……

$$\Rightarrow \quad x = 648.\overline{261} \quad \text{...(i)}$$

Since there are two non-repeating digits (48) on the right hand side of equation (i), therefore we multiply both sides of equation (i) by $(10)^2$ i.e. 100, we get

$$100\,x = 648.\overline{261} \quad \text{...(ii)}$$

Since there are three repeating digits (261), therefore multiply both sides of equation (ii) by $(10)^3$ i.e. 1000, we get

$$100000x = 648261.\overline{261} \quad \text{... (iii)}$$

NOTE: $648.\overline{261}$ can be written as $648.261\overline{261}$ also.

Subtracting equation (ii) from (iii), we get $90900x = 648261 - 648$

$\Rightarrow \quad 90900x = 647613$ ...(iv)

Divide both side of equation (iv) by the coefficient 90900 of x in equation (iv), we get

$$\frac{90900x}{90900} = \frac{647613}{90900} \quad \Rightarrow \quad x = \frac{647613}{90900} \quad \Rightarrow \quad x = \frac{71957}{10100} \text{ (in simplest form)}$$

Hence, $6.48261261261\ldots\ldots\ldots\ldots\ldots\ldots = \frac{71957}{10100}$, which is the required $\frac{p}{q}$ form.

ILLUSTRATION : 4

Express $0.\overline{52}$ in the $\frac{p}{q}$ form.

SOLUTION :

Let $x = 0.\overline{52}$

$x = 0.525252$ ...(i)

There is no non-repeating digit after decimal point on the right hand side in equation (i).

Number of repeating digits after the decimal point on the right hand side of equation (i) is 2. Hence, multiplying both sides of equation (i) by $(10)^2$ i.e. 100, we get

$100\,x = 52.525252.....$ ...(ii)

Subtract (i) from (ii), we get

$$\begin{array}{r} 100x = 52.5252..... \\ x = 0.525252... \\ \hline 99x = 52 \end{array}$$

$$\frac{52}{99} \qquad \therefore \quad 0.\overline{52} = \frac{52}{99}$$

ILLUSTRATION : 5

Express 0.2434343……… in the form of $\frac{p}{q}$.

SOLUTION :

Let $x = 0.2434343\ \ldots\ldots$

$x = 0.2\overline{43}$ ...(i)

Here, a digit 2 does not repeat after decimal point.

So, we multiply equation (i) by 10, we get

$10x = 2.4343$ ...(ii)

Since there are two repeating digits (43) after decimal point in the right side of equation (ii).

So, multiplying (ii) by $(10)^2$ i.e. 100, we get

$1000x = 243.4343$ ...(iii)

Subtract (ii) from (iii), we get

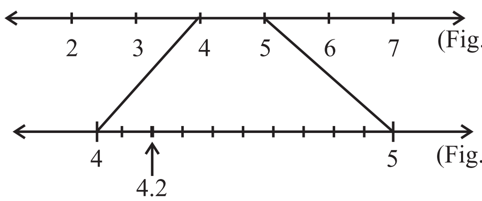
$$\begin{array}{r} 1000x = 243.4343...... \\ 10x = 2.4343...... \\ \hline 990x = 241 \end{array}$$

$$x = \frac{241}{990}$$

$\Rightarrow \quad 0.2434343.......... = \frac{241}{990}$ is the required $\frac{p}{q}$ form.

REPRESENTATION OF DECIMAL NUMBERS ON NUMBER LINE

(i) Representation of 4.2 on the Number Line



(Fig. i)

(Fig. ii)

Clearly 4.2 lies between 4 and 5 in fig. (i).

Take a magnified look of the line segment between 4 and 5 and divide it into 10 equal parts and mark each point of division between 4 and 5 as shown in the fig. (ii).

We can see in fig. (ii), 4.2 is represented by the second mark of division after 4 in between 4 and 5.

(ii) Representation of 4.83 on the Number Line

4.83 will lies in between 4 and 5. In a closer look 4.83 lies in between 4.8 and 4.9 as 4.8 is equal to 4.80 and 4.9 is equal to 4.90 Hence first we draw a number line indicating all integral point on it (fig. (i)). Then after, we find the points on the number line representing 4.8 and 4.9 by dividing the line segment between 4 and 5 in to 10 equal parts using successive magnification (fig. (ii)). Take a magnified look of the line segment between 4.8 and 4.9 and divide it into 10 equal parts. Then third marked point of division shows 4.83 (fig. (iii)).

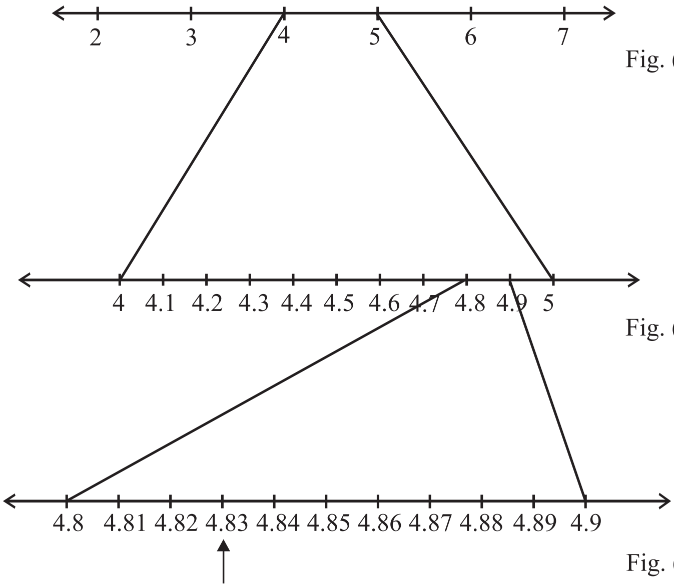


Fig. (i)

Fig. (ii)

Fig. (iii)

(iii) Representation of 6.328 on the Number Line

We know that 6.328 lies between 6 and 7. We divide the portion between 6 and 7 into 10 equal parts as shown in fig (ii) :

Now, 6.328 lies between 6.3 and 6.4, we divide the portion between 6.3 and 6.4 into 10 equal parts as shown in fig (iii) :

Now, 6.328 lies between 6.32 and 6.33, we divide the portion between 6.32 and 6.33 into 10 equal parts as shown in fig (iv) :

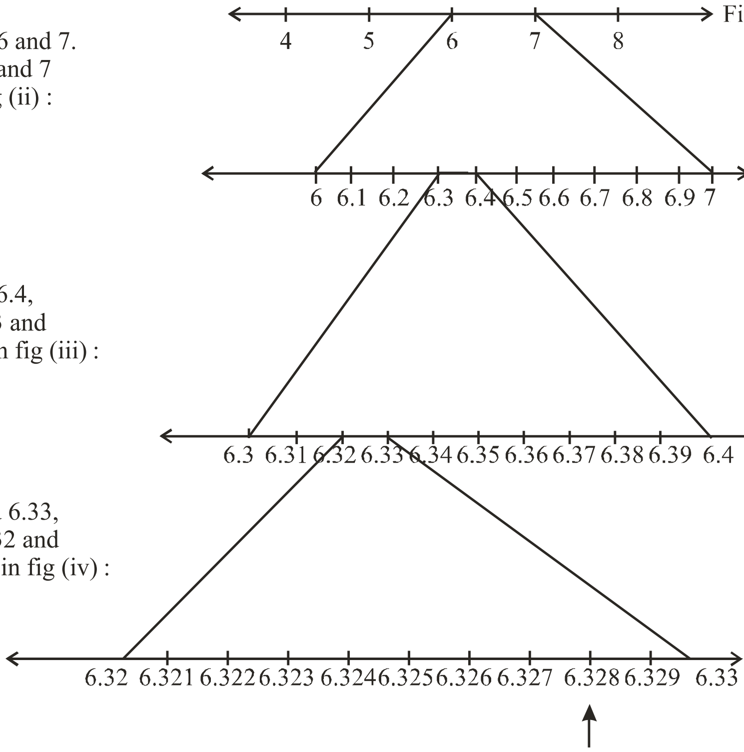


Fig. (i)

Fig. (ii)

Fig. (iii)

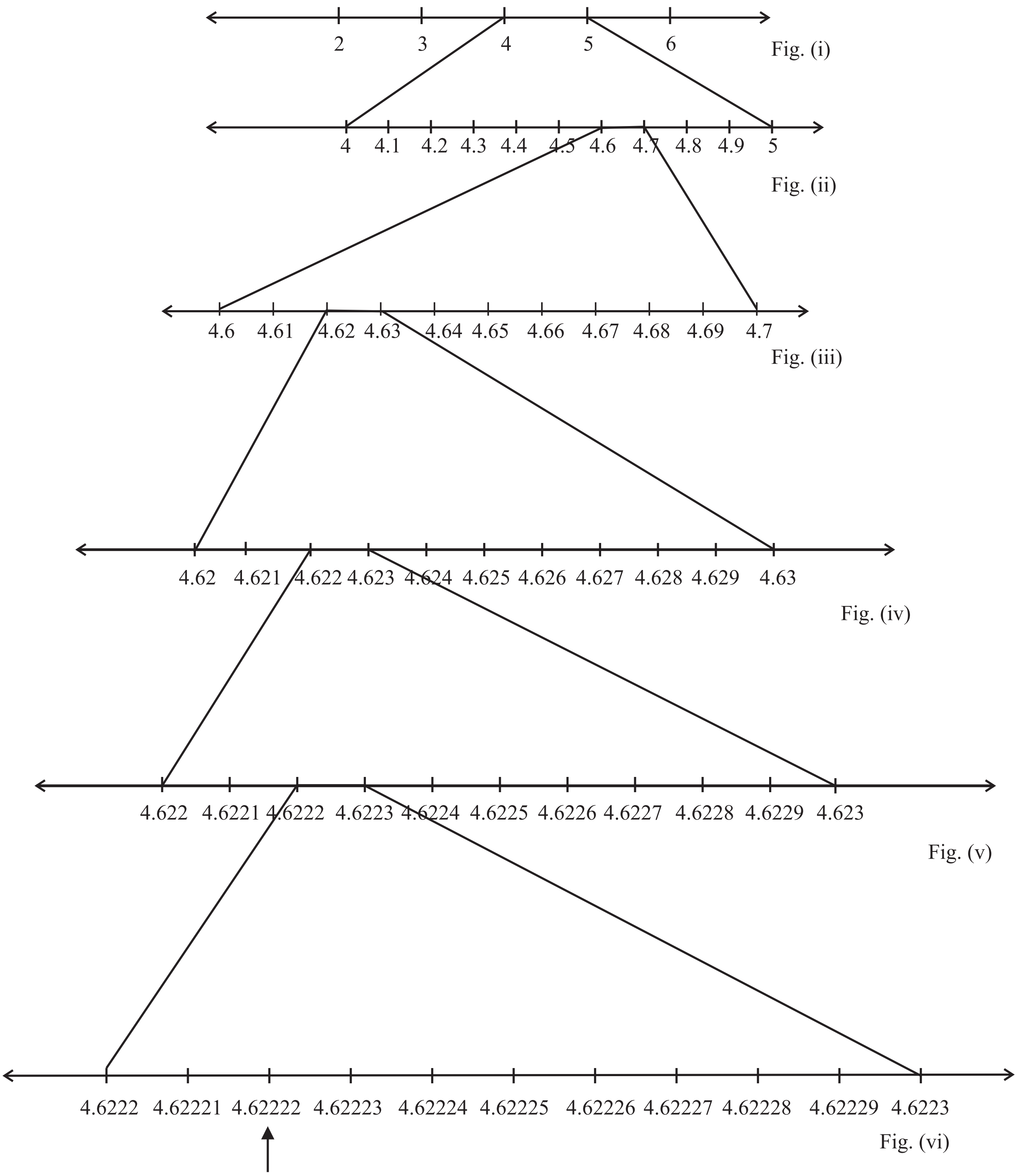
Fig. (iv)

REPRESENTATION OF NON-TERMINATING RECURRING DECIMAL NUMBERS ON NUMBER LINE

Representation of $4.6\overline{2}$ on the Number Line up to Five Decimal Places

$4.6\overline{2} = 4.62222$ (up to 5 decimal places)

we know that $4.6\overline{2}$ lies between 4 and 5. We divide the portion between 4 and 5 into 10 equal parts as shown:



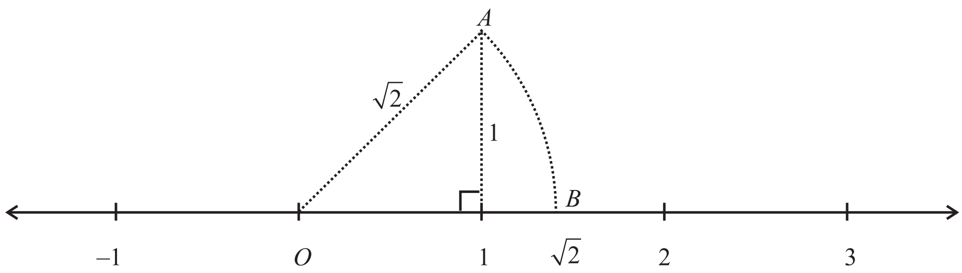
The method of representing a real number in decimal form (up to a certain number of digits) on the number line using the process of magnification is explained above. Now we are able to represent any real number in decimal form (up to a certain number of digits) on the number line using process of successive magnification.

REPRESENTATION OF IRRATIONAL NUMBERS IN THE FORM $\frac{p}{q}$ ON THE NUMBER LINE

To represent an irrational number in the form $\frac{p}{q}$, we use the Pythagoras theorem of a right angle triangle, according to which, in a right angled triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides.
i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{perpendicular})^2$

(i) Representation of $\sqrt{2}$ on the Number Line

Draw a number line indicating all integral points on it.



Since $\sqrt{2} > 0$ therefore, $\sqrt{2}$ lies right side of O on the number line.

Now, $\sqrt{2} = \sqrt{(1)^2 + (1)^2}$

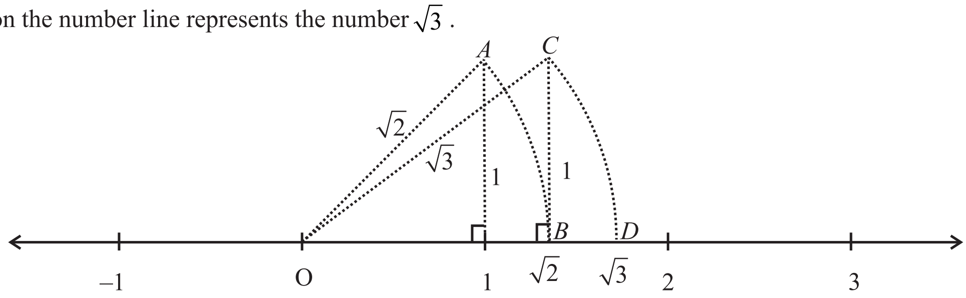
So, if we draw a right angled triangle whose base and perpendicular each is of unit length, then length of its hypotenuse is $\sqrt{2}$ units.
Draw a perpendicular line segment of unit length above the number line at the point representing 1 on it. Let the top-point of this perpendicular line segment be A. Clearly, $OA = \sqrt{2}$ units.
Now draw an arc with O as centre and OB as radius intersecting the number line at a point B on the right side of 1 on the number line. Hence, $OB = \sqrt{2}$ unit.

Therefore point B on the number line represent $\sqrt{2}$.

(ii) Representation of $\sqrt{3}$ on the Number Line

$\sqrt{3} = \sqrt{(\sqrt{2})^2 + 1^2}$

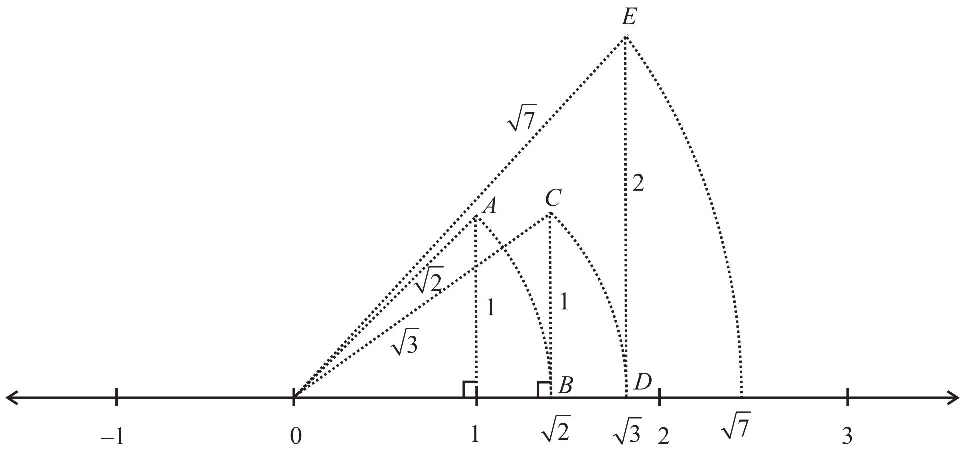
Mark a point B on the number line which represent $\sqrt{\ }$ on the number line as discussed above.
Draw a perpendicular line segment of unit length above the number line at the point representing $\sqrt{2}$ on it.
Let top point of this perpendicular line segment be C. Clearly $OC = \sqrt{3}$ units.
Taking O as centre and OC as radius, draw an arc intersecting the number line at point D on the right side of $\sqrt{2}$ on the number line.
Clearly point D on the number line represents the number $\sqrt{3}$.



(iii) Representation of $\sqrt{7}$ on the Number Line

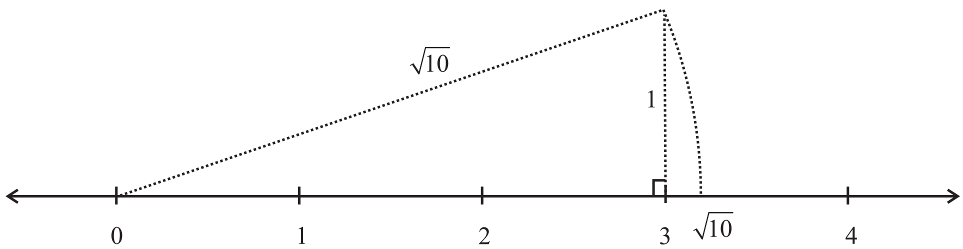
$\sqrt{7} = \sqrt{(\sqrt{6})^2 + 1^2}$ or $\sqrt{7} = \sqrt{(\sqrt{3})^2 + (2)^2}$

Take $\sqrt{7} = \sqrt{\left(\sqrt{3}\right)^2 + (2)^2}$



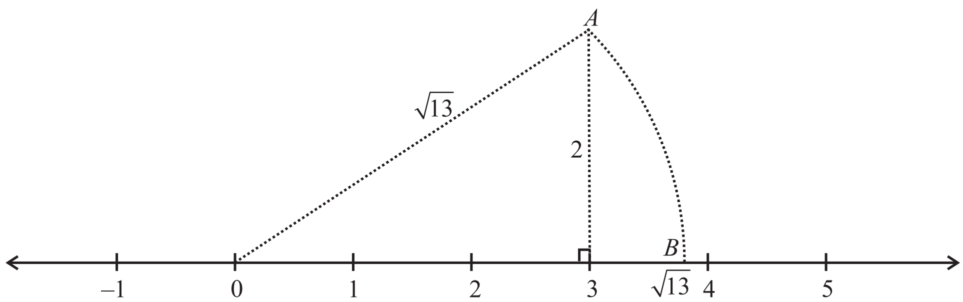
(iv) Representation of $\sqrt{10}$ on the Number Line

$\sqrt{10} = \sqrt{(3)^2 + (1)^2}$



(v) Representation of $\sqrt{13}$ on the Number Line

$\sqrt{13} = \sqrt{3^2 + 2^2}$



TO PROVE $\sqrt{2}$ IS AN IRRATIONAL NUMBER

This is proved by the method of contradiction.

Let us suppose, if possible that $\sqrt{2}$ is not an irrational number i.e. $\sqrt{2}$ is a rational number. Hence, it can be represented in the form of $\frac{p}{q}$ i.e. $\sqrt{2} = \frac{p}{q}$; where p and q are integers but $q \neq 0$.

Let $\frac{p}{q}$ is in simplest form hence p and q are co-primes i.e. they have no common factor other than 1.

Squaring both the sides, we get $2 = \frac{p^2}{q^2}$

$\Rightarrow \quad p^2 = 2q^2 \qquad \text{...(i)}$

Since RHS of (i) is twice the square of an integer, therefore RHS of (i) is even

$\Rightarrow \quad p^2$ is even

Hence p is also an even integer.

Let $p = 2m$, where m is an integer.

$p^2 = 4\,m^2$

Putting the value of p^2 in (i),

$$4\,m^2 = 2\,q^2$$

$$\Rightarrow \quad q^2 = 2m^2 \qquad \text{...(ii)}$$

Since RHS of (ii) is an even integer, hence L.H.S. of it i.e. q^2 is also even integer.

Therefore, q is also an even integer.

Hence, p and q both are even integers.

But two even integers have always a common factor 2, which contradicts our assumption that p and q are co-primes.

Hence, $\sqrt{2}$ is an irrational number.

TO PROVE $(2+\sqrt{3})$ IS AN IRRATIONAL NUMBER

Let us suppose if possible that $2+\sqrt{3}$ is not an irrational number i.e. $2+\sqrt{3}$ is a rational number, then it can be represented in the form of $\frac{p}{q}$ where p and q are integers but $q \neq 0$.

i.e. $\quad 2+\sqrt{3} = \frac{p}{q} \Rightarrow \sqrt{3} = \frac{p}{q} - 2 = \frac{p-2q}{q} \qquad \text{...(i)}$

Since p and q are integer so $(p - 2q)$ will also be, an integer, hence $\frac{p-2q}{q}$ is a rational number.

But $\sqrt{3}$ is an irrational.

This contradicts our assumption that $(2+\sqrt{3})$ is a rational number.

Hence, $(2+\sqrt{3})$ is an irrational number.

TO PROVE $\sqrt{n}$ IS NOT A RATIONAL NUMBER, IF n IS NOT A PERFECT SQUARE

Let n be a whole number but not a perfect square

If possible, let $\sqrt{n}$ be the rational number, then it can be represented in the form of $\frac{p}{q}$, where p and q are integers but $q \neq 0$. Also suppose $\frac{p}{q}$ is in simplest form, hence p and q are co-prime i.e. they have no common factor other than 1.

Now, $\frac{p}{q} = \sqrt{n}$

Squaring both the sides, we get $\frac{p^2}{q^2} = n \Rightarrow \quad p^2 = nq^2 \qquad \text{...(i)}$

Since n is a factor of p^2, therefore, $\quad n$ is a factor of p.

Hence, we can suppose that $p = nm$, where m is an integer.

Putting the value of p in equation (i), we get

$$n^2m^2 = nq^2 \quad \Rightarrow \quad q^2 = nm^2$$

$\therefore \quad n$ is a factor of q^2

$\Rightarrow \quad n$ is a factor of q. $\qquad$ ...(ii)

Hence, n is a factor of both p and q. This contradicts our assumption that p and q does not have any common factor.

This means our assumption that $\sqrt{n}$ is a rational number is wrong. Hence $\sqrt{n}$ is an irrational number, where n is not a perfect square.

PROPERTIES OF IRRATIONAL NUMBERS

(i) Negative of an irrational number is also an irrational number.

For example: $-\sqrt{5}$ is an irrational number, because $\sqrt{5}$ is an irrational number.

(ii) The sum or difference of a rational number and an irrational number is an irrational.

For example : $2+\sqrt{3}$, $2-\sqrt{3}$, $\sqrt{3}-2$ are irrational numbers because 2 is a rational number and $\sqrt{3}$ is an irrational number.

(iii) The product or quotient of a non-zero rational number with an irrational number is an irrational.

For example : $2\sqrt{3}, \frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{2}$ are irrational numbers because 2 is a rational number and $\sqrt{3}$ is an irrational number.

(iv) The sum, difference, product and quotient of two irrational numbers may be rational or irrational.

For examples :

(i) $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two irrational numbers but their sum $= 2+\sqrt{3}+2-\sqrt{3}=4$, is a rational number.

(ii) $(2+\sqrt{3})$ and $(-2+\sqrt{3})$ are two irrational numbers. Their sum $=(2+\sqrt{3})+(-2+\sqrt{3})=2\sqrt{3}$, is an irrational number.

(iii) $(2+\sqrt{3})$ and $(-2+\sqrt{3})$ are two irrational numbers but their difference $=(2+\sqrt{3})-(-2+\sqrt{3})=4$, is a rational number.

(iv) $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two irrational numbers. Their difference $=(2+\sqrt{3})-(2-\sqrt{3})=2\sqrt{3}$ is also an irrational number.

(v) $\sqrt{2}$ and $\sqrt{8}$ are two irrational numbers but their product $=\sqrt{2}\times\sqrt{8}=\sqrt{16}=4$ or -4, is a rational number.

(vi) $\sqrt{2}$ and $\sqrt{3}$ are two irrational numbers. Their product $=\sqrt{2}\times\sqrt{3}=\sqrt{6}$, is an irrational number.

(vii) $\sqrt{8}$ and $\sqrt{2}$ are irrational numbers but their quotient $=\frac{\sqrt{8}}{\sqrt{2}}=\sqrt{\frac{8}{2}}=\sqrt{4}=2$ is a rational number.

(viii) $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are irrational numbers. Their quotient $=\frac{2+\sqrt{3}}{2-\sqrt{3}}=\frac{(2+\sqrt{3})\times(2+\sqrt{3})}{(2-\sqrt{3})\times(2+\sqrt{3})}=\frac{(2+\sqrt{3})^2}{4-3}\ \frac{4+3+4\sqrt{3}}{1}$

$=7+4\sqrt{3}$, is an irrational number.

NOTE: If a is the rational number and n is a positive integer such that the n^{th} root of a is an irrational number, then $a^{1/n}$ is called a surd eg. $\sqrt{5}, \sqrt{2}$ etc. If $\sqrt[n]{a}$ is a surd then 'n' is known as order of surd and 'a' is known as radicand. Every surd is an irrational number but every irrational number, is not a surd.

SOME IDENTITIES RELATED TO SQUARE ROOTS

Let a and b be positive real numbers. Then

(i) $\sqrt{ab}=\sqrt{a}\sqrt{b}$

(ii) $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$

(iv) $(a+\sqrt{b})(a-\sqrt{b})=a^2-b$

(v) $(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d})=\sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$

(vi) $(\sqrt{a}+\sqrt{b})^2=a+2\sqrt{ab}+b.$

LAWS OF RATIONAL EXPONENTS

If a & b are positive real numbers and m & n are rational numbers, then

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^{-n}=\frac{1}{a^n}$

(v) $a^{\frac{m}{n}}=(a^m)^{\frac{1}{n}}=\left(a^{\frac{1}{n}}\right)^m$ i.e. $a^{\frac{m}{n}}=\sqrt[n]{a^m}=(\sqrt[n]{a})^m$

(vi) $(ab)^m = a^m b^m$

(vii) $\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$

ILLUSTRATION : 6

Simplify : $13^{\frac{1}{5}}.17^{\frac{1}{5}}$

SOLUTION :

$$13^{\frac{1}{5}}.17^{\frac{1}{5}}=(13\times17)^{\frac{1}{5}}=221^{\frac{1}{5}}=\sqrt[5]{221}$$

ILLUSTRATION : 7

Simplify: $\left(\dfrac{2^a}{2^b}\right)^{a+b}\left(\dfrac{2^b}{2^c}\right)^{b+c}\left(\dfrac{2^c}{2^a}\right)^{c+a}$

SOLUTION :

$$\left(\frac{2^a}{2^b}\right)^{a+b}\left(\frac{2^b}{2^c}\right)^{b+c}\left(\frac{2^c}{2^a}\right)^{c+a} = (2^{a-b})^{a+b}\cdot(2^{b-c})^{b+c}\cdot(2^{c-a})^{c+a} = 2^{(a^2-b^2)+(b^2-c^2)+(c^2-a^2)} = 2^0 = 1$$

ILLUSTRATION : 8

Prove that $\dfrac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\dfrac{x^b}{x^a}\right)^c = 1$

SOLUTION :

We have,

$$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = \frac{x^{ab-ac}}{x^{ba-bc}} \div (x^{b-a})^c = x^{(ab-ac)-(ba-bc)} \times \frac{1}{x^{(b-a)c}}$$

$$= x^{ab-ac-ba+bc} \times \frac{1}{x^{bc-ac}} = x^{ab-ba-ac+ac+bc-bc} = x^0 = 1$$

RATIONALISING FACTOR

Rationalisation

When the denominator of an expression contains a term with a square root, the procedure of converting it to an equivalent expression whose denominator is a rational number is called rationalising the denominator.

For example :

(i) $(\sqrt{a}+\sqrt{b})$ is the rationalising factor of $(\sqrt{a}-\sqrt{b})$ and vice-versa.

(ii) $(a+\sqrt{b})$ is the rationalising factor of $(a-\sqrt{b})$ and vice-versa.

ILLUSTRATION : 9

Rationalise the denominator of $\dfrac{1}{7+3\sqrt{2}}$

SOLUTION :

We have,

$$\frac{1}{7+3\sqrt{2}} = \frac{1}{7+3\sqrt{2}} \times \frac{7-3\sqrt{2}}{7-3\sqrt{2}} = \frac{7-3\sqrt{2}}{(7)^2 \quad (3\sqrt{2})^2} = \frac{7-3\sqrt{2}}{49-18} = \frac{7-3\sqrt{2}}{31}$$

ILLUSTRATION : 10

Simplify each of the following by rationalising the denominator $\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

SOLUTION:

Multiplying both numerator and denominator by the rationalisation factor of the denominator, we have

$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} \times \frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}} = \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})}$$

$$= \frac{2\sqrt{3}\times2\sqrt{2}-2\sqrt{3}\times3\sqrt{3}-\sqrt{5}\times2\sqrt{2}+\sqrt{5}\times3\sqrt{3}}{(2\sqrt{2})^2-(3\sqrt{3})^2} = \frac{4\sqrt{3\times2}-6\sqrt{3\times3}-2\sqrt{5\times2}+3\sqrt{5\times3}}{4\times2-9\times3}$$

$$= \frac{4\sqrt{6}-6\times3-2\sqrt{10}+3\sqrt{15}}{8-27} = \frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19} = \frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

MISCELLANEOUS SOLVED EXAMPLES

1. Find the value of $2.\overline{6} - 0.\overline{9}$

Sol. Let $x = 2.\overline{6}$ ...(i)

$\therefore \quad 10x = 26.\overline{6}$ ...(ii)

Subtracting (i) from (ii)

$9x = 24$

$\therefore \quad x = \frac{24}{9} = \frac{8}{3}$

Also suppose $\quad y = 0.\overline{9}$ ...(iii)

$\Rightarrow \quad y = 0.9\overline{9}$

$\therefore \quad 10y = 9.\overline{9}$ ...(iv)

Subtracting equation (iii) from (iv), we get

$9y = 9$

$\therefore \quad y = \frac{9}{9} = 1$

$\therefore \quad 2.\overline{6} - 0.\overline{9} = x - y = \frac{8}{3} - 1 = \frac{8-3}{3} = \frac{5}{3}$

2. Show how $\sqrt{5}$ can be represented on the number line.

Sol. Consider a unit square OABC onto the number line with the vertex O which coincides with zero.

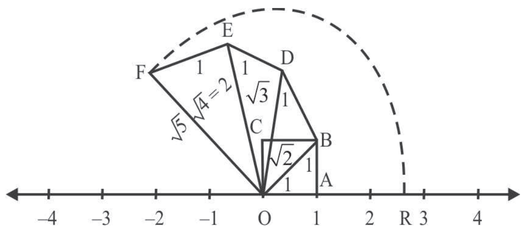
Then OB = $\sqrt{1^2 + 1^2} = \sqrt{2}$

Construct BD of unit length perpendicular to OB. Then OD = $\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$

Construct DE of unit length perpendicular to OD. Then OE = $\sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

Similarly, construct EF of unit length perpendicular to OE. Then OF = $\sqrt{2^2 + 1^2} = \sqrt{5}$

Using a compass, with centre O and radius OF, draw an arc which intersects the number line in the point R. Then R corresponds to $\sqrt{5}$.



Representation of $\sqrt{5}$

3. Rationalise the denominator and simplify : $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

Sol. $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

$$= \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} \times \frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{3\sqrt{2}(\sqrt{3}-\sqrt{6})}{3-6} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-2} + \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{2-3}$$

$$= -\sqrt{2}(\sqrt{3}-\sqrt{6}) - \sqrt{3}(\sqrt{6}-\sqrt{2}) - \sqrt{6}(\sqrt{2}-\sqrt{3}) = -\sqrt{6} + \sqrt{12} - \sqrt{18} + \sqrt{6} - \sqrt{12} + \sqrt{18} = 0$$

4. If $x = \frac{1}{2+\sqrt{3}}$, find the value of $x^3 - x^2 - 11x + 3$

Sol. As $x = \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$ (on rationalizing)

$x - 2 = -\sqrt{3}$

Squaring both sides, we get

$(x-2)^2 = (-\sqrt{3})^2 \Rightarrow x^2 + 4 - 4x = 3 \Rightarrow x^2 - 4x + 1 = 0$

Now, $x^3 - x^2 - 11x + 3 = x^3 - 4x^2 + x + 3x^2 - 12x + 3$

$x(x^2 - 4x + 1) + 3(x^2 - 4x + 1) = x \times 0 + 3(0) = 0 + 0 = 0$

5. Show that $\frac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a x^b x^c)^4} = 1$

Sol. We have $\frac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a x^b x^c)^4}$

$$= \frac{x^{2(a+b)} x^{2(b+c)} x^{2(c+a)}}{(x^a)^4.(x^b)^4(x^c)^4} = \frac{x^{2a+2b}.x^{2b+2c}.x^{2c+2a}}{x^{4a}.x^{4b}.x^{4c}} = \frac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}} = \frac{x^{4a+4b+4c}}{x^{4a+4b+4c}} = 1$$

6. If $\frac{9^n \times 3^2 (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$, prove that $m - n = 1$.

Sol. We have, $\frac{9^n \times 3^2 (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{(3^2)^n \times 3^2 \times 3^{-n/2 \times -2} - (3^3)^n}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27} \quad \Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 2^3} = \frac{1}{27} \Rightarrow \frac{3^{3n}.8}{3^{3m}.8} = \frac{1}{27} \quad \Rightarrow 3^{3n-3m} = \frac{1}{3^3} \quad \Rightarrow 3^{3n-3m} = 3^{-3}$

On equating the exponent, we get

$3n - 3m = -3 \Rightarrow n - m = -1 \Rightarrow m - n = 1.$

7. If $x = 3\sqrt{3} + \sqrt{26}$, then find the value of $\frac{1}{2}\left(x + \frac{1}{x}\right)$

Sol. Let $x = 3\sqrt{3} + \sqrt{26}$

$\frac{1}{x} = \frac{1}{3\sqrt{3}+\sqrt{26}} \times \frac{3\sqrt{3}-\sqrt{26}}{3\sqrt{3}-\sqrt{26}} = \frac{3\sqrt{3}-\sqrt{26}}{(27)-(26)} = 3\sqrt{3} - \sqrt{26}$

$\therefore \frac{1}{2}\left(x + \frac{1}{x}\right) = \frac{1}{2}\left[(3\sqrt{3}+\sqrt{26}) + (3\sqrt{3}-\sqrt{26})\right] = \frac{1}{2} \times 6\sqrt{3} = 3\sqrt{3}$

8. If $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{\frac{1}{4}} = 7^m$, then find the value of m.

Sol. $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{\frac{-1}{3}}\right]^{\frac{1}{4}} = 7^m \quad \Rightarrow \left[\{(7^{-2})^{-2}\}^{-1/3}\right]^{\frac{1}{4}} = 7^m \quad \Rightarrow \left[(7^4)^{-1/3}\right]^{\frac{1}{4}} = 7^m$

$\Rightarrow (7^{-4/3})^{1/4} = 7^m \Rightarrow 7^{-1/3} = 7^m$

$\therefore m = -1/3$

9. Find two irrational numbers between 0.12 and 0.13.

Sol. The two numbers 0.1201001000100001....... and 0.12101001000100001........ are the irrational numbers between 0.12 and 0.13.

10. Find three rational numbers between $\frac{1}{5}$ and $\frac{7}{10}$.

Sol. One rational number between $\frac{1}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{1}{5}+\frac{7}{10}\right) = \frac{1}{2}\left(\frac{2+7}{10}\right) = \frac{9}{20}$

Second rational number between $\frac{1}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{1}{5}+\frac{9}{20}\right) = \frac{1}{2}\left(\frac{4+9}{20}\right) = \frac{13}{40}$

Third rational number between $\frac{1}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{13}{40}+\frac{1}{5}\right) = \frac{1}{2}\left(\frac{13+8}{40}\right) = \frac{21}{80}$

11. Express $1.272727..... = 1.\overline{27}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Sol. Let $x = 1.272727$.......... Since two digits are repeating, we multiply x by 100 to get

$100x = 127.2727$ ....

So, $100x = 126 + 1.272727......... = 126 + x$

Therefore, $100x - x = 126, \Rightarrow 99x = 126 \Rightarrow x = \frac{126}{99} = \frac{14}{11}$

12. Express $0.12\overline{3}$ in $\frac{p}{q}$ form.

Sol. Let $x = 0.12\overline{3}$ i.e. $x = 0.12333$ ... (i)

Multiply both sides of (i) by 100

$100x = 12.333$ ... (ii)

Multiply both sides of (ii) by 10

$1000x = 123.333$ ... (iii)

Subtract (ii) from (iii)

$1000x = 123.333$

$100x = 12.333$

$900x = 111.0$

$\Rightarrow \quad x = \frac{111}{900} = \frac{3\times 37}{900} = \frac{37}{300}$

13. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let us suppose if possible that $\sqrt{3}$ is not an irrational number i.e. $\sqrt{3}$ is a rational number.

Hence $\sqrt{3}$ can be represented in the form of $\frac{p}{q}$.

i.e. $\sqrt{3} = \frac{p}{q}$, where p and q both are integers but $q \neq 0$.

Let $\frac{p}{q}$ is in simplest form, hence p and q are co-prime i.e. they have no common factor other than 1.

Squaring both sides, we get $3 = \frac{p^2}{q^2} \Rightarrow p^2 = 3q^2$ ...(i)

Since q is an integer, therefore q^2 is also an integer.

$\Rightarrow \quad 3q^2$ is an integral multiple of 3.

$\Rightarrow \quad p^2$ is also an integral multiple of 3.

Hence p is also an integral multiple of 3.

Therefore we can assume that $p = 3m$, where m is an integer.

Now putting the value of p in equation (i), we get

$9m^2 = 3q^2, \Rightarrow q^2 = 3m^2 \Rightarrow q^2$ is an integral multiple of 3.

Hence q is an integral multiple of 3.

Since both p and q are integral multiple of 3, therefore they have a common factor 3, which contradict our assumption that p and q are co-primes. Hence $\sqrt{3}$ is an irrational number.

1 EXERCISE

Fill in the Blanks :

DIRECTIONS: *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. The sum/difference of a rational and an irrational number is ______.
2. 0.578 is ______ number. (rational/irrational)
3. 0.72737475 is ______ number. (rational/irrational)
4. If $x+\sqrt{5}=4+\sqrt{y}$, then $x + y$ = ______. (where x and y are rational)
5. An irrational number between $\frac{2}{5}$ and $\frac{3}{7}$ is
6. Value of a is __________ if $\frac{\sqrt{3}-1}{\sqrt{3}+1}=a+b\sqrt{3}$
7. Two mixed quadratic surds, $a+\sqrt{b}$ and $a-\sqrt{b}$, whose sum and product are rational, are called _________.
8. $\sqrt[4]{\frac{1008}{63}}$ is equal to __________
9. Between two rational numbers, there exist __________ number of rational numbers.
10. Between two real numbers, there exists infinite number of __________ numbers.

True / False :

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. Every whole number is a natural number.
2. Every rational number is an integer.
3. Every natural number is a whole number.
4. Every integer is a whole number.
5. Every rational number is a whole number.
6. All rational numbers when expressed in decimal form are either terminating decimals or repeating decimals.
7. All rational numbers can be represented by some point on the number line.
8. The sum or difference of a rational number and an irrational number is an irrational.
9. A real number is either rational or irrational.
10. Product of a rational and an irrational number is always irrational.

Match the Columns :

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column-I have to be matched with statements (p, q, r, s, t) in Column-II.*

1.

	Column-I		Column-II
(A)	12 is a	(p)	prime number
(B)	2,7 are	(q)	not a rational number
(C)	2 is a	(r)	composite number
(D)	$\sqrt{2}$	(s)	co-prime numbers

2.

	Column-I		Column-II
(A)	Decimal expansion of an irrational number is non-terminating and	(p)	irrational
(B)	The sum of two ___ numbers may be a rational number or an irrational number.	(q)	$\sqrt{5}$
(C)	A number x is called a ____ number, if it can be written in the form $\frac{m}{n}$, where m and n are integers, n $\neq$ 0	(r)	rational
(D)	An irrational number between 2 and 2.5 is	(s)	non-repeating
(E)	The value of $0.\overline{23}+0.\overline{22}$ is	(t)	$0.\overline{45}$

Very Short Answer Questions :

DIRECTIONS: *Give answer in one word or one sentence.*

1. Write the smallest whole number.
2. Write the integer other than 1, which is a reciprocal of itself.
3. Suppose a is a rational number. What is the reciprocal of the reciprocal of a ?
4. Write the repeating decimal for each of the following, and use a bar to show the repetend.

   (i) $-\frac{4}{3}$ (ii) $\frac{11}{12}$ (iii) $\frac{7}{13}$

5. Classify the following numbers as rational or irrational.

   (i) $\sqrt{225}$ (ii) 7.478478......

6. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

7. Simplify :

   (i) $\left(\frac{1}{3^3}\right)^7$ (ii) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

8. Simplify :

   (i) $(\sqrt{4})^{3/4}$ (ii) $\left(\frac{5}{8}\right)^3\left(\frac{4}{3}\right)^3$

   (iii) $\left[\dfrac{\frac{\sqrt{5}}{3}}{\frac{5}{9}}\right]^6$ (iv) $\left(\frac{5}{3}\right)^3.\left(\frac{9}{2}\right)^4$

   (v) $\dfrac{2^6 \times 8^2}{4^4}$

9. Find two irrational numbers between 0.1 and 0.2.

10. Determine, without actually dividing, which of the following rational numbers can be named, (a) by a terminating decimal, (b) by a repeating decimal.

    (i) $\frac{7}{20}$ (ii) $\frac{1}{6}$

    (iii) $\frac{1}{12}$ (iv) $3\frac{47}{160}$

11. Write down a fraction which is equivalent to 0.033636363..........

12. Find two rational numbers between 0.222332333233332.... and 0.252552555255552....

13. Which is greatest : $\sqrt[3]{4}$, $\sqrt[4]{5}$ or $\sqrt[4]{3}$?

14. Find four rational numbers between $\frac{1}{4}$ and $\frac{1}{3}$.

Short Answer Questions :

DIRECTIONS: *Give answer in 2-3 sentences.*

1. If $\dfrac{3+\sqrt{5}}{4-2\sqrt{5}} = p + q\sqrt{5}$, where p and q are rational numbers, find the values of p and q.

2. Simplify:

   $$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

3. Examine whether the following numbers are rational or irrational :

   (i) $(2-\sqrt{3})^2$ (ii) $(3+\sqrt{2})(3-\sqrt{2})$

4. Find the value of 'a' in the following expression.

   $$\frac{6}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{2} - a\sqrt{3}$$

5. Simplify : $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

6. Examine whether the number is rational or irrational $\dfrac{(2+\sqrt{2})(3-\sqrt{5})}{(3+\sqrt{5})(2-\sqrt{2})}$.

7. Given that $\sqrt{3} = 1.732$ find the value of $\sqrt{75} + \frac{1}{2}\sqrt{48} - \sqrt{192}$

8. Simplify and express the result in simplest form

   $$\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y} \div \frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}$$

9. Find x^2, if $x = \dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$

Long Answer Questions :

DIRECTIONS: *Give answer in four to five sentences.*

1. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

2. If $x = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

   Then find the value of $x^2 + y^2$.

3. Express with a rational denominator :

   $$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

4. Simplify : $\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

5. Represent $\sqrt{9.3}$ on number line.

6. If $x = \dfrac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$ prove that $b^2x^2 - abx + b^2 = 0$.

7. If $\dfrac{\sqrt{7}-1}{\sqrt{7}+1} - \dfrac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{}$, find the values of a and b

2 EXERCISE

Text-Book Exercise :

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

2. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

3. State whether the following statements are true or false? Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

4. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

5. Are the square roots of all positive integers irrational ? If not, give an example of the square roots of a number that is a rational number.

6. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ (ii) $4\frac{1}{8}$

(iii) $\frac{2}{11}$

7. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[**Hint :** Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

8. Express 0.99999 ..... in the form $\frac{p}{q}$. Are you surprised by your answer?

With your teacher and classmates discuss why the answer makes sense.

9. Write three numbers whose decimal expansions are non-terminating non-recurring.

10. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$ (ii) 7.478478.......

11. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

12. Classify the following numbers as rational or irrational :

(i) $2 - \sqrt{5}$ (ii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iii) 2π.

13. Simplify each of the following expressions :

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$ (ii) $(\sqrt{5} + \sqrt{2})^2$

14. Find : $125^{1/3}$

15. Find :

(i) $32^{2/5}$ (ii) $125^{-1/3}$

16. Simplify :

(i) $2^{\frac{2}{3}} . 2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$

(iii) $\frac{11^{1/2}}{11^{1/4}}$

Exemplar Questions :

1. Simplify the following :

(i) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$

(ii) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

2. Locate $\sqrt{10}$ on the number line.

3. Find the value of:

$$\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{2}{(243)^{\frac{-1}{5}}}$$

4. Rationalise the denominator of the following:

(i) $\frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$ (ii) $\frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}}$

5. Find the value of a in the following:

$$\frac{3 - \sqrt{5}}{3 + 2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}$$

6. If $a = 2 + \sqrt{3}$, then find the value of $a - \frac{1}{a}$

7. Simplify:

(i) $\left(\frac{3}{5}\right)^4 \left(\frac{8}{5}\right)^{-12} \left(\frac{32}{5}\right)^6$

(ii) $\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$

HOTS Questions :

1. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

2. Express 2.5434343… in the form *p/q* where p and q are integers and $q \neq 0$

3. If $x = \frac{1}{7+4\sqrt{3}}, y = \frac{1}{7-4\sqrt{3}}$, find the value of $5x^2 - 7xy - 5y^2$.

4. If $\frac{9^n \times 3^2 (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$, prove that $m - n = 1$.

5. If $a = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $b = \frac{\sqrt{2}-1}{\sqrt{2}+1}$ then find the value of $a^2 + b^2 - 4ab$.

6. If $2^a = 3^b = 6^c$ then show that $c = \frac{ab}{a+b}$.

7. If $x = 2 + 3\sqrt{2}$, then find the value of $\left(x + \frac{14}{x}\right)$.

3 EXERCISE

Single Option Correct :

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The value of x, when $(2)^{x+4}.(3)^{x+1} = 288$ is
(a) 1 (b) –1
(c) 0 (d) None

2. The value of 0.423 is
(a) $\frac{423}{1000}$ (b) $\frac{423}{100}$
(c) $\frac{423}{990}$ (d) $\frac{419}{990}$

3. If $a = 2+\sqrt{3}$ and $b = 2-\sqrt{3}$, then $\frac{1}{a^2} - \frac{1}{b^2}$ is equal to,
(a) 14 (b) – 14
(c) $8\sqrt{3}$ (d) $-8\sqrt{3}$

4. Value of x satisfying $\sqrt{x+3} + \sqrt{x-2} = 5$, is
(a) 6 (b) 7
(c) 8 (d) 9

5. A rational number equivalent to a rational number $\frac{7}{19}$ is
(a) $\frac{17}{119}$ (b) $\frac{14}{57}$
(c) $\frac{21}{38}$ (d) $\frac{21}{57}$

6. Rationalizing factor of $(2+\sqrt{3})$=
(a) $2-\sqrt{3}$ (b) $\sqrt{3}$
(c) $2+\sqrt{3}$ (d) $3+\sqrt{3}$

7. Rationalizing factor of $1+\sqrt{2}+\sqrt{3}$
(a) $1+\sqrt{2}-\sqrt{3}$ (b) 2
(c) 4 (d) $1+\sqrt{2}+\sqrt{3}$

8. Value of $\frac{2^{n+2} - 2(2^n)}{2^{(2n+2)}}$ when simplified is
(a) $1 - 2(2^n)$ (b) $2^{n+3} - \frac{1}{4}$
(c) $\frac{1}{2^{n+1}}$ (d) $\frac{1}{2^{n-1}}$

9. Which of the following statement is not true?
(a) Between two integers, there exist infinite number of rational numbers
(b) Between two rational numbers, there exist infinite number of integers
(c) Between two rational numbers, there exist infinite number of rational numbers
(d) Between two real numbers, there exists infinite number of real numbers

10. Four rational numbers between 3 and 4 are:
(a) $\frac{3}{5}, \frac{4}{5}, 1, \frac{6}{5}$ (b) $\frac{13}{5}, \frac{14}{5}, \frac{16}{5}, \frac{17}{5}$
(c) 3.1, 3.2, 4.1, 4.2 (d) 3.1, 3.2, 3.8, 3.9

More Than One Option Correct :

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following is irrational?
(a) $\sqrt{17}$ (b) $\frac{\sqrt{12}}{\sqrt{3}}$
(c) $\sqrt{7}$ (d) $\sqrt{81}$

2. Value of $\sqrt[4]{(81)^{-2}}$ is

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$

(c) 9 (d) 3

3. Which of the following is equal to x ?

(a) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (b) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$

(c) $(\sqrt{x^3})^{\frac{2}{3}}$ (d) $x^{12/19} + x^{7/19}$

4. Which of the following is/are not correct ?
   (a) Every whole number is a natural number.
   (b) Every integer is a rational number
   (c) Every rational number is an integer
   (d) Every rational number is a whole number

5. Which of the following is/are correct ?
   (a) There are infinitely many rational numbers between any two given rational numbers.
   (b) Every point on the number line represents a unique real number.
   (c) The decimal expansion of an irrational number is non-terminating non-recurring.
   (d) A number whose decimal expansion is non-terminating non-recurring is rational.

Assertion & Reason :

DIRECTIONS: *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason **is not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect**.

(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** Every integer is a rational number
   Reason : Every integer 'm' can be expressed in the form $\frac{m}{1}$.

2. **Assertion :** $\sqrt{2}$ is an irrational number.
   Reason : A number is called irrational, if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

3. **Assertion :** $17^2 . 17^5 = 17^3$
   Reason : If $a > 0$ be a real number and p and q be rational numbers. Then $a^p . a^q = a^{p+q}$.

4. **Assertion :** $2+\sqrt{6}$ is an irrational number.
   Reason : Sum of a rational number and an irrational number is always an irrational number.

5. **Assertion :** A rational number between $\frac{1}{3}$ and $\frac{1}{2}$ is $\frac{5}{12}$.
   Reason : Rational number between two numbers a and b is $\sqrt{ab}$.

Multiple Matching Question :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column-I and four statements (p, q, r, s) in Column-II. Any given statement in Column-I can have correct matching with one or more statement(s) given in Column-II.*

1.

	Column-I		Column-II
(A)	$\frac{1}{2+\sqrt{3}}$	(p)	Irrational
(B)	$64^{1/2}$	(q)	$\sqrt{56}$
(C)	$16^{3/4}$	(r)	8
(D)	$7^{1/2}\, 8^{1/2}$	(s)	$2-\sqrt{3}$

Integer Type Questions :

DIRECTIONS : *Answer the following questions. The answer to each of the question is a single digit integer, ranging from 0 to 9.*

1. If $\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{7}$, find the product of a and b.

2. If $x = \frac{\sqrt{p+2q}+\sqrt{p-2q}}{\sqrt{p+2q}-\sqrt{p-2q}}$ and $q \neq 0$, then find $qx^2 - px + q$

3. If $x = 9+4\sqrt{5}$ and $xy = 1$, then $\frac{1}{322}\left(\frac{1}{x^2}+\frac{1}{y^2}\right)$ is

4. The value of x, if $5^{x-3}.3^{2x-8} = 225$ is

5. If $a^2bc^3 = 25$ and $ab^2 = 5$, then abc equals

6. If $x + 1/x = 3$, then $x^2 + 1/x^2$ is equal to

SOLUTIONS

Brief Explanations of Selected Questions

1 EXERCISE

FILL IN THE BLANKS :

1. irrational 2. rational 3. irrational
4. 9 5. $\sqrt{\frac{6}{35}}$ 6. 2
7. conjugate 8. $\left(\frac{1008}{63}\right)^{1/4} = (16)^{1/4} = (2^4)^{1/4} = 2$
9. infinite 10. real

TRUE/FALSE :

1. False 2. False 3. True 4. False
5. False 6. True 7. True 8. True
9. True 10. True

MATCH THE COLUMNS :

1. (A) → (r); (B) → (s); (C) → (p); (D) → (q);
(A) 12 = 3 × 4 is a composite no.
(B) g.c.d (2,7) = 1
(C) $\sqrt{2}$ is not a rational no.
2. (A) → (s); (B) → (p); (C) → (r); (D) → (q); (E) → (t)

VERY SHORT ANSWER QUESTIONS :

1. 0 2. –1 3. a
4. (i) $-1.\overline{3}$ (ii) $0.91\overline{6}$ (iii) $0.\overline{538461}$
5. (i) rational (ii) rational
6. No, for example $\sqrt{4} = 2$ is a rational number.
7. (i) 3^{-21} (ii) $56^{1/2}$
8. (i) $2^{\frac{3}{4}}$ (ii) $\frac{125}{216}$
(iii) $\frac{729}{125}$ (iv) $\frac{30375}{16}$
(v) 2^4
9. 0.10101001000100001 and 0.11001001000100001
10. (i) terminating (ii) repeating
(iii) repeating (iv) terminating
11. 37/1100 12. 0.25 and 0.2525
13. $\sqrt[3]{4}$ is the greatest 14. $\frac{7}{24}, \frac{13}{48}, \frac{15}{48}, \frac{31}{96}$

SHORT ANSWER QUESTIONS :

1. $p = -11/2$, $q = -5/2$ 2. 5
3. (i) irrational (ii) rational
4. $\frac{6}{3\sqrt{2}-2\sqrt{3}} = \frac{6}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

$= \frac{6(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2} = \frac{6(3\sqrt{2}+2\sqrt{3})}{18-12}$

$= \frac{6(3\sqrt{2}+2\sqrt{3})}{6} = 3\sqrt{2}+2\sqrt{3}$

Therefore, $3\sqrt{2}+2\sqrt{3} = 3\sqrt{2}-a\sqrt{3}$
$\Rightarrow a = -2$

5. $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left((2^3)^{\frac{1}{3}}+(3^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

$= \left[5(2+3)^3\right]^{\frac{1}{4}} = \left[5(5)^3\right]^{\frac{1}{4}} = \left[5^4\right]^{\frac{1}{4}} = 5$

6. On rationalizing the denominator, we get

$\frac{(2+\sqrt{2})(3-\sqrt{5})}{(3+\sqrt{5})(2-\sqrt{2})} = \frac{(2+\sqrt{2})(3-\sqrt{5})}{(3+\sqrt{5})(2-\sqrt{2})} \times \frac{(3-\sqrt{5})(2+\sqrt{2})}{(3-\sqrt{5})(2+\sqrt{2})}$

$= \frac{(2+\sqrt{2})^2(3-\sqrt{5})^2}{(3^2-(\sqrt{5})^2)(2^2-(\sqrt{2})^2)} = \frac{(4+2+4\sqrt{2})(9+5-6\sqrt{5})}{(9-5)(4-2)}$

$= \frac{(6+4\sqrt{2})(14-6\sqrt{5})}{4\times 2} = \frac{84-36\sqrt{5}+56\sqrt{2}-24\sqrt{10}}{8}$

Hence irrational.

7. $\sqrt{75}+\frac{1}{2}\sqrt{48}-\sqrt{192} = 5\sqrt{3}+\frac{4}{2}\sqrt{3}-8\sqrt{3} = \sqrt{3}(5+2-8)$
$= -1.732$

8. $\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y} \times \frac{x-\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}-y} = \frac{x^2-(\sqrt{x^2-y^2})^2}{(\sqrt{x^2+y^2})^2-y^2}$

$= \frac{x^2-(x^2-y^2)}{(x^2+y^2)-y^2} = \frac{y^2}{x^2}$

9. $x^2 = \frac{\sqrt{5}+2+\sqrt{5}-2+2\sqrt{(\sqrt{5})^2-2^2}}{\sqrt{5}+1}$

$= \frac{2\sqrt{5}+2}{\sqrt{5}+1} = \frac{2(\sqrt{5}+1)}{\sqrt{5}+1} = 2$

LONG ANSWER QUESTIONS :

1. $0.6 + 0.\bar{7} + 0.4\bar{7}$

Let $x = 0.\bar{7}\,;\ y = 0.4\bar{7}$

$\Rightarrow\ 9x = 7.\bar{7}\,;\ 10y = 4.\bar{7}$

$\Rightarrow\ 9x = 7\,;\ 100y = 47.\bar{7}$

$\Rightarrow\ x = \dfrac{7}{9}\,;\ 90y = 43 \Rightarrow y = \dfrac{43}{90}$

$\therefore$ Required expression $= \dfrac{6}{10} + \dfrac{7}{9} + \dfrac{43}{90}$

2. 98

3. $\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}$

$= \sqrt{10} + \sqrt{4\times 5} + \sqrt{4\times 10} - \sqrt{5} - \sqrt{16\times 5}$

$= \sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5}$

$= 3\sqrt{10} - 3\sqrt{5} = 3(\sqrt{10} - \sqrt{5})$

$\therefore$ Given expression

$= \dfrac{15}{3(\sqrt{10}-\sqrt{5})} = \dfrac{5}{\sqrt{10}-\sqrt{5}} = \dfrac{5}{\sqrt{10}-\sqrt{5}} \times \dfrac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}$

$= \dfrac{5(\sqrt{10}+\sqrt{5})}{10-5} = \sqrt{10}+\sqrt{5}$

4. Let $I = \dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = A - B - C$

where $A = \dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \dfrac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}} = \dfrac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{10-3}$

$= \dfrac{7\sqrt{30} - 7\times 3}{7} = \dfrac{7(\sqrt{30}-3)}{7} = \sqrt{30} - 3$

$B = \dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \dfrac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \dfrac{2\sqrt{30} - 2\times 5}{6-5} = 2\sqrt{30} - 10$

$C = \dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \dfrac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}} = \dfrac{3\sqrt{30}-18}{15-18} = \dfrac{3\sqrt{30}-18}{-3}$

$= -\sqrt{30} + 6$

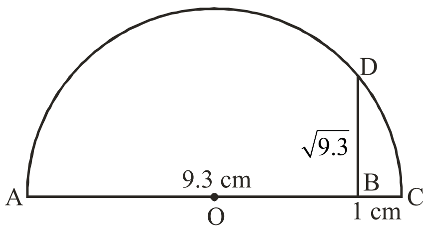
Now, $I = A - B - C$

$= (\sqrt{30}-3) - (2\sqrt{30}-10) - (-\sqrt{30}+6)$

$= \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6$

$= 2\sqrt{30} - 2\sqrt{30} - 3 + 10 - 6 = 1$

5.



Mark a distance 9.3 units from a fixed point A on a given line to obtain a given point B such that AB = 9.3 units. From B mark a distance of 1 unit and call the new point as C. Find the mid point of AC and call that point as O. Draw a semi circle with centre O and radius OC = 5.15 units. Draw a line perpendicular to AC passing through B cutting the semi-circle at D.

Then, BD = $\sqrt{9.3}$ = 3.05 units.

6. $x = \dfrac{(\sqrt{a+2b}+\sqrt{a-2b})}{(\sqrt{a+2b}-\sqrt{a-2b})} \times \dfrac{(\sqrt{a+2b}+\sqrt{a-2b})}{(\sqrt{a+2b}+\sqrt{a-2b})}$

$\Rightarrow\ x = \dfrac{(\sqrt{a+2b}+\sqrt{a-2b})^2}{(a+2b)-(a-2b)}$

$= \dfrac{a+2b+a-2b+2\sqrt{(a+2b)(a-2b)}}{4b}$

$\Rightarrow\ x = \dfrac{\left[a+\sqrt{a^2-4b^2}\right]}{2b}$

$\Rightarrow\ 2bx = a + \sqrt{a^2-4b^2}$

$\Rightarrow\ 2bx - a = \sqrt{a^2-4b^2}$

Squaring both sides, we get

$(2bx-a)^2 = \left(\sqrt{a^2-4b^2}\right)^2$

$\Rightarrow\ 4b^2x^2 + a^2 - 4abx = a^2 - 4b^2$

$\Rightarrow\ 4b^2x^2 - 4abx + 4b^2 = 0$

$\Rightarrow\ b^2x^2 - abx + b^2 = 0$

7. L.H.S. $= \dfrac{\sqrt{7}-1}{\sqrt{7}+1} - \dfrac{\sqrt{7}+1}{\sqrt{7}-1} = \dfrac{(\sqrt{7}-1)^2 - (\sqrt{7}+1)^2}{(\sqrt{7}+1)\,(\sqrt{7}-1)}$

$= \dfrac{(7+1-2\sqrt{7})-(7+1+2\sqrt{7})}{(\sqrt{7})^2-(1)^2}$

$= \dfrac{8-2\sqrt{7}-8-2\sqrt{7}}{7-1}$

$= -\dfrac{4\sqrt{7}}{6} = \dfrac{-2}{3}\sqrt{7}$

$\therefore\ -\dfrac{2}{3}\sqrt{7} = a + b\sqrt{7}$

$\Rightarrow\ 0.a - \dfrac{2}{3}\sqrt{7} = a + b\sqrt{7} \quad \Rightarrow\ a = 0 \text{ and } b = -\dfrac{2}{3}$

2 EXERCISE

TEXT-BOOK EXERCISE :

1. Yes! zero is a rational number. We can write zero in the form $\dfrac{p}{q}$, as follows :

$0 = \dfrac{0}{1} = \dfrac{0}{2} = \dfrac{0}{3}$ ......so on., q can be negative integer also.

2. $\frac{3}{5} = \frac{3 \times 10}{5 \times 10} = \frac{30}{50}$, $\frac{4}{5} = \frac{4 \times 10}{5 \times 10} = \frac{40}{50}$, therefore, five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{31}{50}, \frac{32}{50}, \frac{33}{50}, \frac{34}{50}, \frac{35}{50}$.

3. (i) True, since the collection of whole numbers contains all natural numbers.

(ii) False, because, -3 is not a whole number.

(iii) False, $\because$ $\frac{1}{2}$ is not a whole number.

4. (i) True, ($\because$ real numbers are collection of rational and irrational numbers.)

(ii) False

(iii) False, (2 is real but not irrational.)

5. No, the square roots of all positive integers are not irrational. For example, $\sqrt{16} = 4$ is a rational number.

6. (i) Given $\frac{36}{100}$ = 0.36. Hence, the decimal expansion is terminating.

(ii) Consider $4\frac{1}{8} = 4 + \frac{1}{8} = \frac{33}{8}$

$\therefore$

$$\begin{array}{r l}
8\,)\;33.000\;(4.125 & \\
\underline{32} & \\
10 & \\
\underline{8} & \\
20 & \\
\underline{16} & \\
40 & \\
\underline{40} & \\
\times &
\end{array}$$

Thus, from the divison, we get $4\frac{1}{8} = 4.125$

Hence, The decimal expansion of $4\frac{1}{8}$ is terminating.

(iii) Consider $\frac{2}{11}$

$$\begin{array}{r l}
11\,)\;2.0000\;(0.1818...... & \\
\underline{11} & \\
90 & \\
\underline{88} & \\
20 & \\
\underline{11} & \\
90 & \\
\underline{88} & \\
2 &
\end{array}$$

On dividing by 11, we get

$\frac{2}{11} = 0.1818..... = 0.\overline{18}$.

So, the decimal expansion of $\frac{2}{11}$ is non-terminating repeating.

7. Yes! we can predict the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ without actually doing the long division as follows :

$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = 0.\overline{428571}$

Similarly,

$\frac{4}{7} = 0.\overline{571428}$

$\frac{5}{7} = 0.\overline{714285}$

$\frac{6}{7} = 0.\overline{857142}$.

8. Let x = 0.99999........

Multiplying both sides by 10 (since one digit is repeating), we get

$10x = 9.9999.....$

$\Rightarrow\ 10x = 9 + 0.99999.....\qquad \Rightarrow\ 10x = 9 + x$

$\Rightarrow\ x = \frac{9}{9} = 1$

Thus, 0.99999.....= 1 = $\frac{1}{1}$

Which is in the form $\frac{p}{q}$,

where, $p = 1$

$q = 1$ $(q \neq 0)$

Since 0.99999........goes on for ever, so there is no gap between 1 and 0.99999...... end. Hence they are equal.

9. 0.01001 0001 00001........,

0.20 2002 20003 200002........,

0.003000300003.............,

10. (i) $\sqrt{23}$

	4.795831523
4	23.00 00 00 00 00 00 00 00 00
	16
87	700
	609
949	9100
	8541
9585	55900
	47925
95908	797500
	767264
959163	3023600
	2877489
9591661	14611100
	9591661
95916625	501943900
	479583125
959166302	2236077500
	1918332604
9591663043	31774489600
	28774989129
	2999500471

Thus, $\sqrt{23} = 4.795831523........$

$\sqrt{23}$ is an irrational number.

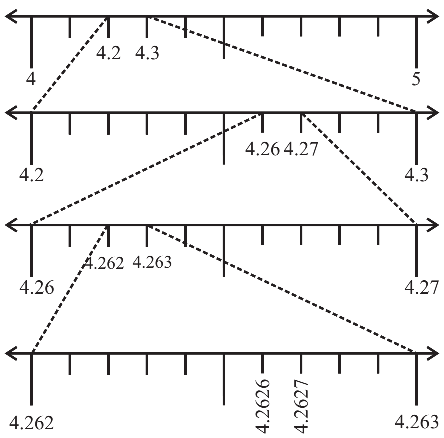
$\therefore$ The decimal expansion is non-terminating non-recurring.

(ii) $7.478478.....= 7.\overline{478}$

Since, the decimal expansion is non- terminating recurring.

$\therefore$ 7.478478......... is a irrational number.

11. $4.\overline{26} = 4.262626......$



12. (i) $2 - \sqrt{5}$ is an irrational number as 2 is a rational number and $\sqrt{5}$ is an irrational number.

($\because$ The difference of a rational number and an irrational number is irrational.)

(ii) Consider

$$\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$$

which is a rational number.

$$\left(\frac{p}{q}, q \neq 0 \text{ form}\right)$$

where $p = 2$, $q = 7$ ($\neq 0$)

(iii) Since, the product of a non-zero rational number with an irrational number is irrational.

$\therefore$ 2π is an irrational number.

13. (i) Consider

$(3 + \sqrt{3})(2 + \sqrt{2})$

$= (3)(2) + 3\sqrt{2} + (\sqrt{3})(2) + (\sqrt{3})(\sqrt{2})$

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{(3)(2)}$ $\left(\because \sqrt{a}\sqrt{b} = \sqrt{ab}\right)$

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$.

(ii) Consider

$(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + 2\sqrt{5}\ \sqrt{2} + (\sqrt{2})^2$

$= 5 + 2\sqrt{10} + 2$ $\left(\because \sqrt{a}\sqrt{b} = \sqrt{ab}\right)$

$= 7 + 2\sqrt{10}$.

14. Consider $125^{1/3} = (5^3)^{1/3} = 5^{3\times 1/3} = 5^1 = 5$.

15. (i) Consider $32^{2/5} = (2^5)^{2/5} = 2^{5\times 2/5} = 2^2 = 4$.

(ii) Consider $125^{-1/3} = (5^3)^{-1/3} = 5^{3\times(-1/3)} = 5^{-1} = \frac{1}{5}$

16. (i) Consider $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = 2^{2/3 + 1/5}$

$= 2^{\frac{10+3}{15}} = 2^{13/15}$.

(ii) Consider $\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{(3^3)^7} = \frac{1}{3^{21}} = 3^{-21}$

(iii) Consider $\frac{11^{1/2}}{11^{1/4}} = 11^{1/2-1/4} = 11^{1/4}$.

EXEMPLAR QUESTIONS :

1. (i) $3\sqrt{3} + 2\sqrt{3\times3\times3} + \frac{7}{\sqrt{3}}$

$= 3\sqrt{3} + 2\sqrt{3\times3\times3} + \frac{7}{\sqrt{3}}$

$= 3\sqrt{3} + 6\sqrt{3} + \frac{7}{\sqrt{3}}$

$= \frac{3\sqrt{3}\times\sqrt{3} + 6\sqrt{3}\times\sqrt{3} + 7}{\sqrt{3}}$

$= \frac{9+18+7}{\sqrt{3}} = \frac{34}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{34\sqrt{3}}{3}$

(ii) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

$= \sqrt[4]{3\times3\times3\times3} - 8\sqrt[3]{6\times6\times6}$

$+ 15\sqrt[5]{2\times2\times2\times2\times2} + \sqrt{15\times15}$

$= 3 - 8 \times 6 + 15 \times 2 + 15$

$= 3 - 48 + 30 + 15$

$= -45 + 45$

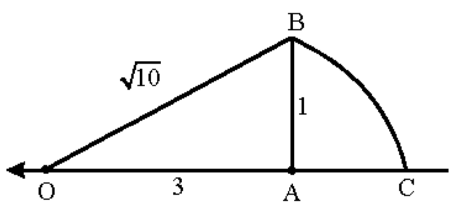
$= 0$

2. We write 10 as the sum of the squares of two natural numbers.

$10 = 9 + 1 = 3^2 + 1^2$

Take OA = 3 units, on the number line

Draw BA = 1 unit, perpendicular to OA. Join OB



Now, by Pythagoras theorem.

$OB^2 = AB^2 + OA^2$

$OB^2 = 1^2 + 3^2 = 10$

$\Rightarrow$ $OB = \sqrt{10}$

Taking O as centre and OB as a radius, draw an arc which intersects the number line at point C. Clearly, C corresponds to $\sqrt{10}$ on the number line.

3. $\dfrac{4}{(216)^{\frac{-2}{3}}}+\dfrac{1}{(256)^{\frac{-3}{4}}}+\dfrac{2}{(243)^{\frac{-1}{5}}}$

$=4(216)^{\frac{2}{3}}+(256)^{\frac{3}{4}}+2(243)^{\frac{1}{5}}$

$=4\left(6^3\right)^{\frac{2}{3}}+\left(4^4\right)^{\frac{3}{4}}+2\left(3^5\right)^{\frac{1}{5}}$

$=4\times 6^{\frac{6}{3}}+4^{\frac{12}{4}}+2\times 3^{\frac{5}{5}}=4\times 6^2+4^3+2\times 3$

$= 4 \times 36 + 64 + 6 = 144 + 64 + 6 = 214$

4. (i) $\dfrac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\dfrac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$=\dfrac{3\sqrt{5}\left(\sqrt{5}+\sqrt{3}\right)+\sqrt{3}\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$

$=\dfrac{3\times 5+3\sqrt{5}\times\sqrt{3}+\sqrt{3}\times\sqrt{5}+\sqrt{3}\times\sqrt{3}}{5-3}$

$=\dfrac{15+3\sqrt{15}+\sqrt{15}+3}{2}$

$=\dfrac{18+4\sqrt{15}}{2}=9+2\sqrt{15}$

(ii) $\dfrac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}=\dfrac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}\times\dfrac{\sqrt{48}-\sqrt{18}}{\sqrt{48}-\sqrt{18}}$

$=\dfrac{4\sqrt{3}\times\sqrt{48}+5\sqrt{2}\times\sqrt{48}-4\sqrt{3}\times\sqrt{18}-5\sqrt{2}\times\sqrt{18}}{\left(\sqrt{48}\right)^2-\left(\sqrt{18}\right)^2}$

$=\dfrac{4\sqrt{3\times3\times4\times4}+5\sqrt{2\times2\times2\times2\times2\times3}-4\sqrt{3\times3\times6}-5\sqrt{2\times3\times3\times2}}{48-18}$

$=\dfrac{4\times3\times4+5\times2\times2\sqrt{6}-4\times3\sqrt{6}-5\times3\times2}{30}$

$=\dfrac{48-30+20\sqrt{6}-12\sqrt{6}}{30}$

$=\dfrac{18+8\sqrt{6}}{30}=\dfrac{9+4\sqrt{6}}{15}$

5. We have $\dfrac{3-\sqrt{5}}{3+2\sqrt{5}}=a\sqrt{5}-\dfrac{19}{11}$

Consider,

$\dfrac{3-\sqrt{5}}{3+2\sqrt{5}}=\dfrac{3-\sqrt{5}}{3+2\sqrt{5}}\times\dfrac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$=\dfrac{9-3\sqrt{5}-6\sqrt{5}+10}{(3)^2-\left(2\sqrt{5}\right)^2}$

$=\dfrac{19-9\sqrt{5}}{9-20}=\dfrac{19-9\sqrt{5}}{-11}$

$=-\dfrac{19}{11}+\dfrac{9}{11}\sqrt{5}=a\sqrt{5}-\dfrac{19}{11}$

Comparing the coefficient of $\sqrt{5}$, we get

$a=\dfrac{9}{11}$

6. We have, $a=2+\sqrt{3}$

$\dfrac{1}{a}=\dfrac{1}{2+\sqrt{3}}$

$=\dfrac{2-\sqrt{3}}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}$

$=\dfrac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$

$=\dfrac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$

$\therefore\quad a-\dfrac{1}{a}=2+\sqrt{3}-\left(2-\sqrt{3}\right)$

$2+\sqrt{3}-2+\sqrt{3}=2\sqrt{3}$

$\therefore\ a-\dfrac{1}{a}=2\sqrt{3}$

7. (i) $\left(\dfrac{3}{5}\right)^4\left(\dfrac{8}{5}\right)^{-12}\left(\dfrac{32}{5}\right)^6$

$=\dfrac{3^4\times 8^{-12}\times 32^6}{5^4\times 5^{-12}\times 5^6}=\dfrac{3^4\times(2)^{-36}\times(2)^{30}}{5^{4-12+6}}$

$=\dfrac{3^4\times(2)^{-36+30}}{5^{-2}}=\dfrac{3\times3\times3\times3\times2^{-6}}{5^{-2}}$

$=\dfrac{81\times5\times5}{2\times2\times2\times2\times2\times2}=\dfrac{2025}{64}$

(ii) $\dfrac{9^{\frac{1}{3}}\times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}}\times 3^{-\frac{2}{3}}}=\dfrac{(3)^{2\times\frac{1}{3}}\times(3)^{3\times-\frac{1}{2}}}{(3)^{\frac{1}{6}}\times 3^{-\frac{2}{3}}}$

$=\dfrac{3^{\frac{2}{3}}\times 3^{\frac{-3}{2}}}{3^{\frac{1}{6}}\times 3^{\frac{-2}{3}}}=\dfrac{3^{\frac{2}{3}-\frac{3}{2}}}{3^{\frac{1}{6}-\frac{2}{3}}}=\dfrac{3^{\frac{-5}{6}}}{3^{\frac{-1}{2}}}$

$=3^{\frac{-5}{6}+\frac{1}{2}}=3^{-\frac{1}{3}}$

HOTS QUESTIONS :

1. Consider the rational number $\frac{5}{7}$. On dividing by 7, we get,

$$\begin{array}{r} 7\,)\,5.000000\,(0.714285\ldots\ldots \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 5 \end{array}$$

Thus, $\frac{5}{7} = 0.714285 \ldots\ldots = 0.\overline{714285}$

Now, consider $\frac{9}{11}$.

On dividing by 11, we get

$$\begin{array}{r} 11)\,9.0000\,(0.8181\ldots\ldots \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 9 \end{array}$$

Thus, $\frac{9}{11} = 0.8181\ldots\ldots = 0.\overline{81}$

Three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$ can be

0.75 075007500075000075.......,
0.7670767000767............ and
0.808008000800008...........

2. Let $x = 2.5434343$

$\Rightarrow x = 2.5\overline{43}$

Multiplying both sides by 10 we get

$10x = 25.\overline{43}$ … (i)

Again multiplying equation (i) by 100

$1000x = 2543.\overline{43}$

On subtracting

$1000x - 10x = 2543.\overline{43} - 25.\overline{43}$

$990x = 2518$

$x = \frac{2518}{990}$

$x = \frac{1259}{495}$

Hence, $2543.\overline{43} - 25.\overline{43} = \frac{1259}{495}$

3. $-7\left[1 + 80\sqrt{3}\right]$

4. We have $\frac{9^n \times 3^2 (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{(3^2)^n \times 3^2 \times 3^{-n/2 \times -2} - (3^3)^n}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 2^3} = \frac{1}{27} \Rightarrow \frac{3^{3n}.8}{3^{3m}.8} = \frac{1}{27}$

$\Rightarrow 3^{3n-3m} = \frac{1}{3^3}$

$\Rightarrow 3^{3n-3m} = 3^{-3}$

On equating the exponent, we get

$\Rightarrow 3n - 3m = -3 \Rightarrow n - m = -1 \Rightarrow m - n = 1.$

5. Here, $a = \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2 - 1^2}$

$\frac{(\sqrt{2})^2 + 1 + 2\sqrt{2}}{2-1} = \frac{2+1+2\sqrt{2}}{1} = 3 + 2\sqrt{2}$

$\therefore a = 3 + 2\sqrt{2}$ ...(i)

$b = \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2 - 1}$

$= \frac{(\sqrt{2})^2 + 1^2 - 2\sqrt{2}}{2-1} = \frac{2+1-2\sqrt{2}}{1} = 3 - 2\sqrt{2}$

$\therefore b = 3 - 2\sqrt{2}$ ... (ii)

From equations (i) and (ii)

$a + b = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$

$ab = (3 + 2\sqrt{2})\,(3 - 2\sqrt{2}) = 3^2 - \left(2\sqrt{2}\right)^2$

$= 9 - 4 \times 2 = 9 - 8 = 1$

$a^2 + b^2 - 4ab = a^2 + b^2 + 2ab - 6ab$

$= (a + b)^2 - 6ab$

$= 6^2 - 6 \times 1 = 36 - 6 = 30.$

6. Let $2^a = 3^b = 6^c = k$

So, $2^a = k$

$\Rightarrow k^{1/a} = 2$ …(i)

Similarly, $k^{1/b} = 3$ …(ii)

$k^{1/c} = 6$ …(iii)

Now, we know that $6 = 2 \times 3$

Putting the values of 6, 2 and 3 from (i), (ii) and (iii), we get:

$k^{1/c} = k^{1/a} \times k^{1/b} \Rightarrow k^{1/c} = k^{1/a + 1/b}$

$$\Rightarrow \frac{1}{c} = \frac{1}{a} + \frac{1}{b} \Rightarrow \frac{1}{c} = \frac{b+a}{ab} \Rightarrow c = \frac{ab}{a+b}$$

7. $x = 2 + 3\sqrt{2}$

$$\frac{1}{x} = \frac{1}{2+3\sqrt{2}}$$

By rationalising, we get:

$$\frac{1}{x} = \frac{2-3\sqrt{2}}{2^2 - \left(3\sqrt{2}\right)^2} = \frac{2-3\sqrt{2}}{4-18} = \frac{3\sqrt{2}-2}{14}$$

Now,

$$\left(x + \frac{14}{x}\right) = 2 + 3\sqrt{2} + 14\left(\frac{3\sqrt{2}-2}{14}\right)$$

$$= 2 + 3\sqrt{2} + 3\sqrt{2} - 2$$

$$= 6\sqrt{2}$$

3 EXERCISE

SINGLE OPTION CORRECT :

1. (a) $2^{x+4} . 3^{x+1} = 2^5 . 3^2 \Rightarrow x = 1$

2. (a) **3.** (d)

4. (a) $x = 6$ satisfies the given equation

5. **(d)** Simplest form of $\frac{17}{119} = \frac{17}{119}$

Simplest form of $\frac{14}{57} = \frac{14}{57}$

Simplest form of $\frac{21}{38} = \frac{21}{38}$

Simplest form of $\frac{21}{57} = \frac{7}{19}$

6. (a) **7.** (a) **8.** (c) **9.** (b)

10. (d) To find four rational numbers between 3 and 4.

$\frac{3\times5}{5}$ and $\frac{4\times5}{5}$

$\Rightarrow \frac{15}{5}$ and $\frac{20}{5}$

Between $\frac{15}{5}$ and $\frac{20}{5}$ lies $\frac{16}{5}, \frac{17}{5}, \frac{18}{5}, \frac{19}{5}$

Now, from the given options (a) and (b) does not contain rational number between 3 and 5.

(c) has 4.1 and 4.2 that does not lie between 3 and 4.

MORE THAN ONE OPTION CORRECT :

1. (a) and (c) **2.** (b)

3. (c) **4.** (a), (c) and (d)

5. (a), (b) and (c)

ASSERTION AND REASON :

1. (a)

2. (a)

3. (d) $17^2 . 17^5 = 17^{2+5} = 17^7$

4. (a)

5. (c) $\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{5}{12}$

MULTIPLE MATCHING QUESTIONS :

1. (A) $\to$ (p, s); (B) $\to$ (r) ; (C) $\to$ (r); (D) $\to$ (q)

INTEGER TYPE QUESTIONS :

1. **Ans : 0**

$$\text{L.H.S.} = \frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = \frac{(\sqrt{7}-1)^2 - (\sqrt{7}+1)^2}{(\sqrt{7}+1)(\sqrt{7}-1)}$$

$$= \frac{(7+1-2\sqrt{7}) - (7+1+2\sqrt{7})}{(\sqrt{7})^2 - (1)^2}$$

$$= \frac{8-2\sqrt{7}-8-2\sqrt{7}}{7-1}$$

$$= -\frac{4\sqrt{7}}{6} = -\frac{2}{3}\sqrt{7}$$

$$\therefore \quad -\frac{2}{3}\sqrt{7} = a + b\sqrt{7}$$

$\Rightarrow a = 0$ and $b = -\frac{2}{3}$

$\Rightarrow a - b = 0$

2. **Ans : 0**

$$\frac{x}{1} = \frac{\sqrt{p+2q}+\sqrt{p-2q}}{\sqrt{p+2q}-\sqrt{p-2q}}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt{p+2q}}{2\sqrt{p-2q}} \quad \text{(By componendo and dividendo)}$$

On squaring both sides, we get

$$\frac{(x^2+1)+2x}{(x^2+1)-2x} = \frac{p+2q}{p-2q}$$

On again applying componendo and dividendo.

we have,

$$\frac{2(x^2+1)}{2.2x} = \frac{2.p}{2.2q}$$

$$qx^2 + q = px$$

Then, $qx^2 - px + q = 0$.

3. Ans : 1

$x = 9 + 4\sqrt{5}$

$y = \frac{1}{x} = \frac{1}{9+4\sqrt{5}} = \frac{9-4\sqrt{5}}{(9)^2-(4\sqrt{5})^2} = 9-4\sqrt{5}$

$\therefore \quad \frac{1}{322}\left(\frac{1}{x^2}+\frac{1}{y^2}\right)$

$= \frac{1}{322}[(9-4\sqrt{5})^2+(9+4\sqrt{5})^2]$

$= \frac{2(81+80)}{322} = \frac{2(161)}{322} = \frac{322}{322} = 1$

4. Ans : 5

$5^{x-3}\,3^{2x-8} = 225 \Rightarrow 5^{x-3}\,3^{2x-8} = 5^2.3^2$

By comparing $x-3=2 \Rightarrow x = 3+2 = 5.$

5. Ans : 5

$a^2bc^3 = 25$ ......(i)

$ab^2 = 5$ ......(ii)

Multiplying (i) & (ii), we get $a^3\,b^3\,c^3 = 125$

$abc = (125)^{1/3} = 5$

6. Ans : 7

$x + \frac{1}{x} = 3$ (given)

Squaring both sides $x^2 + \frac{1}{x^2} + 2 = 9$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 7.$