CHAPTER

7

INTEGRALS

EXERCISE 1: NCERT BASED TOPIC-WISE MCQS

7.1 & 7.2 INTRODUCTION, INTEGRATION AS AN INVERSE PROCESS OF DIFFERENTIATION

1. $\int x^{51}(\tan^{-1} x + \cot^{-1} x)dx$ NCERT Page-296/N-226

(a) $\frac{x^{52}}{52}(\tan^{-1} x + \cot^{-1} x) + c$

(b) $\frac{x^{52}}{52}(\tan^{-1} x - \cot^{-1} x) + c$

(c) $\frac{\pi x^{52}}{104} + \frac{\pi}{2} + c$

(d) $\frac{x^{52}}{52} + \frac{\pi}{2} + c$

2. $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx =$ NCERT Page-295/N-226

(a) $e.3^{-3x} + c$

(b) $e^3 \log x + c$

(c) $\frac{x^3}{3} + c$

(d) None of these

3. Evaluate : $\int (x^2 + x)^2 dx$ NCERT Page-296/N-229

(a) $\frac{x^5}{5} + \frac{x^3}{3} + \frac{x^4}{2}$

(c) $5x^5 + 3x^5 + 8x^4$

(b) $\frac{x^5}{5} + \frac{x^3}{3} + \frac{x^4}{2} + c$

(d) $5x^5 + 3x^3 + 8x^4 + c$

4. Evaluate: $\int \frac{x^2-5x+6}{x-2} dx$ NCERT Page-296/N-228

(a) $\frac{x^2}{2} - 3x$

(b) $x - 3$

(c) $\frac{x^2}{2} - 3x + c$

(d) $\frac{x^2}{2} + 3x + c$

5. Evaluate: $\int \frac{x^2+x+1}{\sqrt{x}} dx$ NCERT Page-296/N-226

(a) $\frac{5}{2}x^{5/2} + \frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + c$

(b) $\frac{2}{5}x^{5/2} + \frac{3}{2}x^{3/2} + 2x^{1/2} + c$

(c) $x^{3/2} + x^{1/2} + x^{-1/2}$

(d) $\frac{3}{2}x^{1/2}+\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{-3/2}+c$

6. $\int \frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$

NCERT Page-295/N-226

(a) $\tan x+\cot x+c$
(b) $\operatorname{cosec} x+\operatorname{sce} x+c$
(c) $\tan x+\sec x+c$
(d) $\tan x+\operatorname{cosec} x+c$

7.3 METHODS OF INTEGRATION

7. For $\mathrm{I}(x)=\int \frac{\sec^2 x-2022}{\sin^{2022} x}dx$, if $\mathrm{I}\left(\frac{\pi}{4}\right)$, then
(a) $3^{1010}\mathrm{I}\left(\frac{\pi}{3}\right)-\mathrm{I}\left(\frac{\pi}{6}\right)=0$
(b) $3^{1010}\mathrm{I}\left(\frac{\pi}{6}\right)-\mathrm{I}\left(\frac{\pi}{3}\right)=0$.
(c) $3^{1011}\mathrm{I}\left(\frac{\pi}{3}\right)-\mathrm{I}\left(\frac{\pi}{6}\right)=0$
(d) $3^{1011}\mathrm{I}\left(\frac{\pi}{6}\right)-\mathrm{I}\left(\frac{\pi}{3}\right)=0$

8. $\int x^x(1+\log x)dx$ is equal to

NCERT Page-350/N-236

(a) x^x+C
(c) $\mathrm{x}^{\mathrm{x}}\log \mathrm{x}+\mathrm{C}$
(b) $\mathrm{x}^{2\mathrm{x}}+\mathrm{C}$
(d) $1/2(1+\log x)^2+C$

9. If the integral $\int \frac{\cos 8x+1}{\cot 2x-\tan 2x}dx=A\cos 8x+k$, where k is an arbitrary constant, then A is equal to :

NCERT Page-301/N-236

(a) $-\frac{1}{16}$
(b) $\frac{1}{16}$
(c) $\frac{1}{8}$
(d) $-\frac{1}{8}$

10. Let $\int \frac{x^{1/2}}{\sqrt{1-\mathrm{x}^3}}dx=\frac{2}{3}\operatorname{gof}(\mathrm{x})+C$, then

NCERT Page-304/N-239

(a) $f(x)=\sqrt{x}$
(b) $f(x)=x^{3/2}$ and $g(x)=\sin^{-1} x$
(c) $f(x)=x^{2/3}$
(d) None of these

11. $\int \sec^{2/3} x\operatorname{cosec}^{4/3} x dx=$

NCERT Page-303/N-238

(a) $-3(\tan x)^{1/3}+c$
(c) $3(\tan x)^{-1/3}+c$
(b) $-3(\tan x)^{-1/3}+c$
(d) $(\tan x)^{-1/3}+c$

12. $\int \frac{dx}{\cos x+\sqrt{3}\sin x}$ equals

NCERT Page-304/N-238

(a) $\log \tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(b) $\log\,\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\frac{1}{2}\log\,\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(d) $\frac{1}{2}\log\,\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

13. Let $g:(0,\infty)\rightarrow R$ be a differentiable function such that $\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{xg(x)}{e^x+1}+c$, for all $x>0$, where c is an arbitrary constant. Then NCERT Page-325/N-236

(a) g is decreasing in $\left(0,\frac{\pi}{4}\right)$

(b) g' is increasing in $\left(0,\frac{\pi}{4}\right)$

(c) $g+g'$ is increasing in $\left(0,\frac{\pi}{2}\right)$

(d) ' $g-g'$ is increasing in $\left(0,\frac{\pi}{2}\right)$

14. $\int\frac{(x^2+1)e^x}{(x+1)^2}dx=f(x)e^x+C$, where C is a constant, then $\frac{d^3f}{dx^3}$ at $x=1$ is equal to:

NCERT Page-326/N-236

(a) $-\frac{3}{4}$

(b) $\frac{3}{4}$

(c) $-\frac{3}{2}$

(d) $\frac{3}{2}$

15. The integral $\int\frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to NCERT Page-304/N-238

(a) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(b) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(c) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(d) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

16. $\int\frac{1}{[(x-1)^3(x+2)^5]^{1/4}}dx$ is equal to NCERT Page-304/N-238

(a) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$

(b) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4}+C$

(c) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$

(d) $\frac{1}{3}\left(\frac{x+2}{x-1}\right)^{1/4}+C$

17. $\int\frac{2dx}{(e^x+e^{-x})^2}=$ NCERT Page-304/N-238

(a) $\frac{-e^{-x}}{(e^x+e^{-x})}+C$

(b) $\frac{-1}{(e^x+e^{-x})}+C$

(c) $\frac{1}{(e^x+1)^2}+C$

(d) $\frac{1}{(e^x+e^{-x})}+C$

18. If $\int x^{13/2}\cdot\left(1+x^{5/2}\right)^{1/2}dx = A\left(1+x^{5/2}\right)^{7/2}+B\left(1+x^{5/2}\right)^{5/2}+C\left(1+x^{5/2}\right)^{3/2}+D$, then

(a) $\mathrm{A}=-\frac{4}{35}, \mathrm{B}=-\frac{8}{25}, \mathrm{C}=\frac{4}{15}$

(b) $\mathrm{A}=\frac{4}{35}, \mathrm{B}=-\frac{8}{25}, \mathrm{C}=-\frac{4}{15}$

(c) $A=\frac{4}{35}, B=-\frac{8}{25}, C=\frac{4}{15}$

(d) None of these

19. $\int(27e^{9x}+e^{12x})^{1/3}dx$ is equal to NCERT Page-304/N-237

(a) $(1/4)(27+\mathrm{e}^{3\mathrm{x}})^{1/3}+C$

(b) $(1/4)(27+\mathrm{e}^{3\mathrm{x}})^{2/3}+\mathrm{C}$

(c) $(1/3)(27+e^{3x})^{4/3}+C$

(d) $(1/4)(27+e^{3x})^{4/3}+C$

20. $\int\frac{e^x(1+x)}{\cos^2(e^x x)}dx$ equals

(a) $-\cot(ex^x)+C$ NCERT Page-304/N-239

(b) $\tan(xe^x)+C$

(c) $\tan(e^x)+C$

(d) $\cot(\mathrm{e}^{\mathrm{x}})+\mathrm{C}$

21. Evaluate: $\int 2^{2^{2^x}}2^{2^x}2^x dx$ NCERT Page-304/N-239

(a) $\frac{1}{(\log 2)^3}2^{2^{2^x}}+C$

(b) $\frac{1}{(\log 2)^3}2^{2^x}+C$

(c) $\frac{1}{(\log 2)^2}2^{2^x}+C$

(d) $\frac{1}{(\log 2)^4}2^{2^{2x}}+C$

22. $\int\frac{10\mathrm{x}^9+10^{\mathrm{x}}\log_{\mathrm{e}}10}{10^{\mathrm{x}}+\mathrm{x}^{10}}\mathrm{dx}$ is equal to

(a) $10^x-x^{10}+C$ NCERT Page-304/N-236

(b) $10^x+x^{10}+C$

(c) $(10^x-x^{10})^{-1}+C$

(d) $\log_e(\log^x+x^{10})+C$

23. $\int\cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is equal to NCERT Page-304/N-237

(a) $\frac{1}{8}(\mathrm{x}^2-1)+\mathrm{k}$

(b) $\frac{1}{2}x^2+k$

(c) $\frac{1}{2}x+k$

(d) None of these

24. $\int\mathrm{e}^{3\log x}(\mathrm{x}^4+1)^{-1}\mathrm{dx}$ is equal to NCERT Page-305/N-237

(a) $\log(\mathrm{x}^4+1)+\mathrm{C}$

(b) $\frac{1}{4}\log(\mathrm{x}^4+1)+\mathrm{C}$

(c) $-\log\left(x^4+1\right)+C$
(d) None of these

25. If $\int \sin^3 x\cos^5 x dx = A\sin^4 x + B\sin^6 x + C\sin^8 x + D$. Then NCERT Page-303/N-242
(a) $A=\frac{1}{4}, B=-\frac{1}{3}, C=\frac{1}{8}, D \in R$
(b) $A=\frac{1}{8}, B=\frac{1}{4}, C=\frac{1}{3}, D \in R$
(c) $A=0, B=-\frac{1}{6}, C=\frac{1}{8}, D \in R$
(d) None of these

26. $\int \left(x+\frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right) dx$ is equal to : NCERT Page-303/N-242
(a) $\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$
(b) $\left[\frac{x^2+1}{x^2}\right]^{n+6}(n+6)+c$
(c) $\left[\frac{x}{x^2+1}\right]^{n+6}(n+6)+c$
(d) None of these

27. $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x} dx$ is equal to NCERT Page-304/N-242
(a) $\frac{1}{2}\sin 2x + C$
(b) $-\frac{1}{2}\sin 2x + C$
(c) $-\frac{1}{2}\sin x + C$
(d) $-\sin^2 x + C$

28. If $\int \cos^n x\sin x dx = -\frac{\cos^6 x}{6} + C$, then $n=$
NCERT Page-303/N-242
(a) 0
(b) 1
(c) 2
(d) 5

29. Evaluate: $\int \sin^3 x\cos^3 x dx$ NCERT Page-304/N-242
(a) $\frac{1}{32}\left\{\frac{3}{2}\cos 2x-\frac{1}{6}\cos 6x\right\}+C$
(b) $\frac{1}{32}\left\{-\frac{3}{2}\cos 2x+\frac{1}{6}\cos 6x\right\}+C$
(c) $\frac{1}{32}\left\{-\frac{3}{2}\cos 2x-\frac{1}{6}\cos 6x\right\}+C$
(d) None of these

30. Evaluate: $\int \frac{1}{\sqrt{\sin^3 x\cos^5 x}} dx$ NCERT Page-303/N-242
(a) $\frac{2}{\sqrt{\tan x}}-\frac{2}{3}(\tan x)^{3/2}+C$
(b) $-\frac{2}{\sqrt{\tan x}}+\frac{2}{3}(\tan x)^{3/2}+C$
(c) $-\frac{2}{\sqrt{\tan x}}-\frac{2}{3}(\tan x)^{3/2}+C$
(d) None of these

31. $\int \frac{1}{x\log x\log(\log x)}dx =$

(a) $\log\log(x) + c$ NCERT Page-303/N-238

(b) $\log[\log(\log x)] + c$

(c) $-\log\log(\log x) + c$

(d) none of these

32. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log\sin(x-\alpha) + C$, then value of (A, B) is

(a) $(-\cos\alpha, \sin\alpha)$ NCERT Page-303/N-242

(b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$

(d) $(\sin\alpha, \cos\alpha)$

33. $\int \frac{x^2\tan^{-1} x^3}{1+x^6}dx$ is equal to NCERT Page-303/N-237

(a) $\tan^{-1} x^3 + C$

(b) $\frac{1}{6}(\tan^{-1} x^3)^2 + C$

(c) $-\frac{1}{2}(\tan^{-1} x^3)^2 + \mathrm{C}$

(d) $\frac{1}{2}(\tan^{-1} x^2)^3 + C$

34. Value of $\int \frac{(x-x^3)^{1/3}}{x^4}dx$ is NCERT Page-304/N-237

(a) $\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}} + \mathrm{C}$

(b) $\frac{-3}{8}\left(\frac{1}{x^2}-1\right)^{\frac{4}{3}} + C$

(c) $\frac{-3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}} + C$

(d) None of these

35. The value of $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$ is NCERT/ Page-304/N-237

(a) $\frac{1}{2}\log|e^x + e^{-x}| + C$

(b) $2\log|e^{2x} + e^{-2x}| + C$

(c) $\frac{1}{2}\log|e^{2x} + e^{-2x}| + C$

(d) None of these

7.4 INTEGRALS OF SOME PARTICULAR FUNCTIONS

36. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c, g(1) = 0$, then $g\left(\frac{1}{2}\right)$ is equal to: NCERT Page-314/N-244

(a) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + \frac{\pi}{3}$

(b) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{3}$

(c) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) - \frac{\pi}{3}$

(d) $\frac{1}{3}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) - \frac{\pi}{6}$

37. Evaluate: $= \int \frac{\tan x \cdot \sec x}{\sqrt{\sec^2 x+4}} dx$

NCERT Page-314/N-244

(a) $\log\left|\sec x - \sqrt{\sec^2 x + 4}\right| + c$

(b) $\log\left|\sec x + \sqrt{\sec^2 x + 4}\right| + c$

(c) $\log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + c$

(d) $\frac{1}{2}\tan^{-1}\left(\frac{\sec x}{2}\right) + c$

38. Evaluate: $\int \frac{\mathrm{x}}{1-\mathrm{x}^4} \mathrm{dx}$

NCERT Page-315/N-250

(a) $\frac{1}{4}\log\left|\frac{1+\mathrm{x}^2}{1-\mathrm{x}^2}\right| + \mathrm{c}$

(b) $\frac{1}{4}\log\left|\frac{1-\mathrm{x}^2}{1+\mathrm{x}^2}\right| + \mathrm{c}$

(c) $\frac{1}{2}\log\left|\frac{1+\mathrm{x}^2}{1-\mathrm{x}^2}\right| + \mathrm{c}$

(d) $\frac{1}{2}\log\left|\frac{1-\mathrm{x}^2}{1+\mathrm{x}^2}\right| + \mathrm{c}$

39. Value of $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ if $(\beta > \alpha)$ is

NCERT Page-314/N-251

(a) $2\sin^{-1}\sqrt{\frac{\beta-\alpha}{x-\alpha}} + C$

(b) $2\sin^{-1}\sqrt{\frac{x-\alpha}{\beta-\alpha}} + C$

(c) $2\sin^{-1}\sqrt{\frac{x+\alpha}{\beta-\alpha}} + C$

(d) None of these

40. Evaluate: $\int \frac{x^3+x}{x^4-9} dx$

NCERT Page-314/N-250

(a) $\frac{1}{4}\log|\mathrm{x}^4 - 9| + \frac{1}{12}\log\left|\frac{\mathrm{x}^2+3}{\mathrm{x}^2-3}\right| + C$

(b) $\frac{1}{4}\log|\mathrm{x}^4 - 9| - \frac{1}{12}\log\left|\frac{\mathrm{x}^2-3}{\mathrm{x}^2+3}\right| + C$

(c) $\frac{1}{4}\log|x^4 - 9| + \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right| + C$

(d) None of these

41. Evaluate: $\int \frac{1}{\sqrt{9+8x-x^2}} dx$

NCERT Page-314/N-249

(a) $-\sin^{-1}\left(\frac{x-4}{5}\right) + C$

(b) $-\sin^{-1}\left(\frac{x+4}{5}\right) + C$

(c) $\sin^{-1}\left(\frac{x-4}{5}\right) + C$

(d) None of these

42. The value of $\int_0^\pi \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})} dx$ is equal to

NCERT Page-336/N-251

(a) $\frac{\pi^2}{4}$

(b) $\frac{\pi^2}{2}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

43. $\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})} dx$ is equal to NCERT Page-312/N-251

(a) $\frac{3}{2}x^{2/3} + 6\tan^{-1} x^{1/6} + C$

(b) $\frac{3}{2}\mathrm{x}^{2/3} - 6\tan^{-1} \mathrm{x}^{1/6} + \mathrm{C}$

(c) $-\frac{3}{2}\mathrm{x}^{2/3} + 6\tan^{-1} \mathrm{x}^{1/6} + \mathrm{C}$

(d) None of these

44. Evaluate: $\int \frac{1}{1+3\sin^2 x+8\cos^2 x} dx$

(a) $\frac{1}{6}\tan^{-1}(2\tan \mathrm{x}) + \mathrm{C}$ NCERT Page-313/N-245

(b) $\tan^{-1}(2\tan x) + C$

(c) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(d) None of these

45. Value of $\int \frac{x^2+1}{x^4+x^2+1} dx$ is NCERT Page-313/N-245

(a) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{x^2-1}\right) + C$

(b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) + C$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{3}x}\right) + C$

(d) None of these

46. Value of $\int \frac{dx}{\sqrt{x(a-x)}}$ is NCERT/ Page-314/N-246

(a) $2\sin^{-1}\sqrt{\frac{x}{a}} + c$

(b) $-2\sin^{-1}\sqrt{\frac{x}{a}} + c$

(c) $2\sin^{-1}\frac{\sqrt{\mathrm{x}}}{\mathrm{a}} + \mathrm{c}$

(d) None of these

47. Value of $\int \frac{dx}{4\sin^2 x+4\sin x\cos x+5\cos^2 x}$ is NCERT Page-314/N-245

(a) $\frac{-1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right) + C$

(b) $\frac{1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right) + C$

(c) $\frac{1}{22}\tan^{-1}\left(\frac{\tan x+2}{2}\right) + C$

(d) None of these

48. If $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx = A\sqrt{\mathrm{x}^2 - 9\mathrm{x} + 20} + \mathrm{B}\log\left|\mathrm{x} + \sqrt{\mathrm{x}^2 - 9\mathrm{x} + 20} - \frac{9}{2}\right| + \mathrm{C}$

Then the values of A and B are NCERT Page-313/N-247

(a) 6,34

(c) 12,17

(b) 3,9

(d) None of these

49. $\int \frac{dx}{x\sqrt{1-x^3}} = a\ln\left(\frac{\sqrt{1-x^3}+b}{\sqrt{1-x^3}+1}\right) + k$, then:

(a) b=1, a = 1 NCERT Page-313/247

(b) $b = -1$, and $a = \frac{1}{3}$

(c) $b = 1, a = -\frac{2}{3}$

(d) $b = 1$ and $a = -\frac{1}{3}$

50. $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$ equal to NCERT Page-314/N-247

(a) $\frac{1}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}} + C$

(b) $\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}} + C$

(c) $\frac{2}{3}\sin^{-1}\sqrt{\frac{x}{a}} + C$

(d) None of these

51. The value of $\int \sqrt{\frac{a-x}{a+x}}dx$ is NCERT Page-315/N-239

(a) $a\sin^{-1}\left(\frac{x}{a}\right) + \sqrt{x^2 - a^2} + C$

(b) $a\sin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2 - x^2} + C$

(c) $a\sin^{-1}\left(\frac{a}{x}\right) + \sqrt{x^2 - a^2} + C$

(d) None of these

52. If $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{3} + c$, then $a =$ NCERT Page-315/N-244

(a) 3

(b) 4

(c) 6

(d) 8

53. $\int \sqrt{4x^2+9}dx$ is equal to NCERT Page-325/N-244

(a) $\frac{x}{2}\sqrt{4x^2+9} - \frac{9}{4}\log\left|2x + \sqrt{4x^2+9}\right| + c$

(b) $\frac{x}{2}\sqrt{4x^2+9} + \frac{9}{4}\log\left|2x + \sqrt{4x^2+9}\right| + c$

(c) $-\frac{x}{2}\sqrt{4x^2+9} - \frac{9}{4}\log\left|2x + \sqrt{4x^2+9}\right| + c$

(d) $\frac{x}{2}\sqrt{4x^2+9} + \frac{9}{4}\sin^{-1}(4x^2+9)\Big| \quad + c$

54. Evaluate: $\int \sqrt{7x - 10 - x^2}dx$ NCERT Page-324/N-244

(a) $\frac{1}{4}(2x-7)\sqrt{7x-10-x^2} + \frac{9}{8}\sin^{-1}\left(\frac{2x-7}{3}\right) + c$

(b) $-\frac{1}{4}(2x-7)\sqrt{7x-10-x^2} - \frac{9}{8}\sin^{-1}\left(\frac{2x-7}{3}\right) + c$

(c) $-\frac{1}{4}(2x-7)\sqrt{7x-10-x^2} + \frac{9}{8}\sin^{-1}\left(\frac{2x-3}{7}\right) + c$

(d) None of these.

55. $\int \sqrt{x^2 - 8x + 7}dx =$ NCERT Page-326/N-244

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} + 9\log x - 4 + \sqrt{x^2-8x+7} + c$

(b) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left[x-4+\sqrt{x^2-8x+7}\right]+c$
(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left[x-4+\sqrt{x^2-8x+7}\right]+c$
(d) None of these

7.5 INTEGRATION BY PARTIAL FRACTIONS

56. $\int\left(\frac{ax+b}{cx+d}\right)dx=\frac{3x}{2}-\log\sqrt{x+2}+k$, Then $\frac{\mathrm{ab}}{\mathrm{cd}}=$ NCERT Page-315/N-254
(a) $\frac{2}{7}$
(b) $\frac{15}{8}$
(c) $\frac{3}{8}$
(d) $\frac{8}{15}$

57. $\int\frac{x}{(x-1)(x^2+4)}dx$. is equal to NCERT Page-313/N-256
(a) $\frac{1}{5}\log|x-1|-\frac{1}{10}\log(x^2+4)+\frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right)+C$
(b) $\frac{2}{5}\log|x-1|+\log(x^2+4)-\tan^{-1}\left(\frac{x}{2}\right)+C$
(c) $\frac{4}{5}(\log|x-1)+\log(x^2+4)-2\tan^{-1}\left(\frac{x}{2}\right)+C$
(d) None of these

58. Integration of $\int\frac{dx}{(x-1)(x+2)(2x+3)}$ is NCERT Page-314/N-254
(a) $\log|x+2|-\frac{1}{3}\log|x-1|+2\log|2x+3|+C$
(b) $\frac{1}{15}\log|x-1|+\frac{1}{3}\log|x+2|-\frac{2}{5}\log|2x+3|+C$
(c) $\frac{4}{5}\log|x-1|-\frac{1}{15}\log(x-1)-\frac{1}{3}\log|x+2|+C$
(d) None of these

59. If $\int\frac{3x+1}{(x-3)(x-5)}dx=\int\frac{-5}{(x-3)}dx+\int\frac{B}{(x-5)}dx$, then the value of B is NCERT Page-314/N-254
(a) 3
(b) 4
(c) 6
(d) 8

60. Evaluate: $\int\frac{1}{(x-1)(x^2+2x+1)}dx$ NCERT Page-314/N-255
(a) $\frac{1}{4}\log\left|\frac{x+1}{x-1}\right|+\frac{1}{2(x+1)}+c$
(b) $\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+\frac{1}{4(x+1)}+c$
(c) $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|+\frac{1}{2(x+1)}+c$
(d) $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2(x+1)}+c$

61. Value of $\int\frac{x^2+1}{(x-1)(x-2)}dx$ is NCERT Page-313/N-255
(a) $x+\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$ (b) $x+\log\left[\frac{(x-1)^2}{(x-2)^5}\right]+C$

(c) $x-\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$ (d) None of these

62. If $\int \frac{1}{(\sin x+4)(\sin x-1)} dx$

$= \mathrm{A}\frac{1}{\left(\tan\frac{x}{2}-1\right)}+\mathrm{B}\tan^{-1}[\mathrm{f(x)}]+\mathrm{C}_1$, then NCERT Page-314/N-254

(a) $\mathrm{A}=-\frac{1}{5}, \mathrm{B}=\frac{-2}{5\sqrt{15}}, \mathrm{f(x)}=\frac{4\tan x+3}{\sqrt{15}}$

(b) $A=-\frac{1}{5}, B=\frac{1}{\sqrt{15}}, f(x)=\frac{4\tan\left(\frac{x}{2}\right)+1}{\sqrt{15}}$

(c) $\mathrm{A}=\frac{2}{5}, \mathrm{B}=-\frac{2}{5}, \mathrm{f(x)}=\frac{4\tan x+1}{5}$

(d) $A=\frac{2}{5}, B=\frac{-2}{5\sqrt{15}}, f(x)=\frac{4\tan\left(\frac{x}{2}\right)+f}{\sqrt{15}}$

63. Evaluate: $\int \frac{1-\cos x}{\cos x(1+\cos x)} dx$ NCERT Page-314/N-256

(a) $\log|\sec x+\tan x|-2\tan(x/2)+C$

(b) $\log|\sec x-\tan x|-2\tan(x/2)+C$

(c) $\log|\sec x+\tan x|+2\tan(x/2)+C$

(d) None of these

64. If $\int \frac{3x+4}{x^3-2x-4} dx=\log|x-2|+k\log f(x)+c$, then

(a) $f(x)=|x^2+2x+2|$

(b) $f(x)=x^2+2x+2$

(c) $\mathrm{k}=-\frac{1}{2}$

(d) All of these

65. $\int \frac{x^3-1}{x^3+x} dx$ is equal to

(a) $x-\log x+\log(x^2+1)-\tan^{-1} x+C$ NCERT Page-315/N-255

(b) $x-\log x+\frac{1}{2}\log(x^2+1)-\tan^{-1} x+C$

(c) $x+\log x+\frac{1}{2}\log(x^2+1)+\tan^{-1} x+C$

(d) None of these

7.6 INTEGRATION BY PARTS

66. If $\int x\log\left(1+\frac{1}{x}\right) dx=f(x)\log(x+1)+g(x)x^2+Lx+C$, then NCERT Page-324/N-261

(a) $f(x)=\frac{1}{2}x^2$

(b) $g(x)=\log x$

(c) $\mathrm{L}=1$

(d) None of these

67. $\int \tan^{-1}\sqrt{\mathrm{x}}\mathrm{dx}$ is equal to NCERT Page-325/N-260

(a) $(\mathrm{x}+1)\tan^{-1}\sqrt{\mathrm{x}}-\sqrt{\mathrm{x}}+\mathrm{C}$

(b) $x\tan^{-1}\sqrt{x}-\sqrt{x}+C$

(c) $\sqrt{x}-x\tan^{-1}\sqrt{x}+C$

(d) $\sqrt{x}-(x+1)\tan^{-1}\sqrt{x}+C$

68. The value of $\int \cos(\log x) dx$ is : NCERT Page-325/N-261

(a) $\frac{1}{2}[\sin(\log x)+\cos(\log x)]+C$
(b) $\frac{x}{2}[\sin(\log x)+\cos(\log x)]+C$
(c) $\frac{x}{2}[\sin(\log x)-\cos(\log x)]+C$
(d) $\frac{1}{2}[\sin(\log x)-\cos(\log x)]+C$

69. If $\int \frac{e^x(1+\sin x)dx}{1+\cos x}=e^x f(x)+C$, then $f(x)$ is equal to NCERT Page-324/N-262
(a) $\sin\frac{x}{2}$
(b) $\cos\frac{x}{2}$
(c) $\tan\frac{x}{2}$
(d) $\log\frac{x}{2}$

70. $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ is equal to NCERT Page-324/N-262
(a) $-e^x\tan\left(\frac{x}{2}\right)+C$
(b) $-e^x\cot\left(\frac{x}{2}\right)+C$
(c) $-\frac{1}{2}e^x\tan\left(\frac{x}{2}\right)+C$
(d) $\frac{1}{2}e^x\cot\left(\frac{x}{2}\right)+C$

71. $\int \sin^{-1} x dx$ is equal to NCERT Page-325/N-260
(a) $x\sin^{-1} x+\sqrt{1-x^2}+c$ (b)
$x\sin^{-1} x-\sqrt{1-x^2}+c$
(c) $\cos^{-1} x+c$
(d) $\frac{1}{\sqrt{1-x^2}}+c$

72. Find $\int \frac{x\sin^{-1} x}{\sqrt{1-x^2}}dx$.
(a) $x+\sqrt{1-x^2}\sin^{-1} x+c(b)$ NCERT Page-327/N-261
(b) $x-\sqrt{1-x^2}\sin^{-1} x+c$
(c) $\sqrt{1-x^2}+\sin^{-1} x+c$
(d) $\sqrt{1-x^2}-\sin^{-1} x+c$

73. Find $\int xe^x dx$ NCERTPage-325/N-263
(a) $e^x(x-1)+c$
(b) $e^x(x+1)+c$
(c) $x(e^x-1)+c$
(d) $x(e^x+1)+c$

74. Find $\int \log x dx$ NCERT Page-324/N-261
(a) $x(\log x+1)+c$
(b) $x(\log x-1)+c$
(c) $\log x+x+c$
(d) $\frac{1}{x}+c$

75. $\int x\sin x\sec^3 x dx =$ NCERT Page-326/N-262

(a) $\frac{1}{2}[\sec^2 x - \tan x] + c$

(b) $\frac{1}{2}[x\sec^2 x - \tan x] + c$

(c) $\frac{1}{2}[x\sec^2 x + \tan x] + c$

(d) $\frac{1}{2}[\sec^2 x + \tan x] + c$

76. $\int x\sec^2 x dx =$ NCERT Page-326/N-261

(a) $\tan x + \log \cos x + c$

(b) $x\tan x + \log \sec x + c$

(c) $x\tan x + \log \cos x + c$

(d) $\frac{x^2}{2}\sec^2 x + \log \cos x + c$

77. $\int e^x(1 + \tan x + \tan^2 x)dx =$ NCERT Page-325/N-263

(a) $e^x\sin x + c$

(c) $e^x\tan x + c$

(b) $e^x\cos x + c$

(d) $e^x\sec x + c$

78. $\int x^3 e^{ax} dx$ is NCERT Page-324/N-261

(a) $\frac{1}{a^3}e^{ax}(3a^2x^2 - 6ax + 6) + c$

(b) $\frac{1}{a^4}e^{ax}(a^3x^3 - 3a^2x^2 + 6ax - 6) + c$

(c) $\frac{1}{a^5}e^{ax}(a^4x^4 - 5a^3x^3 + 3a^2x^2 - 6ax + 6) + c$

(d) None of these

79. $\int \sin 2x \cdot \log \cos x dx$ is equal to NCERT Page-324/N-260

(a) $\cos^2 x\left(\frac{1}{2} + \log \cos x\right) + C$

(b) $\cos^2 x \cdot \log \cos x + C$

(c) $\cos^2 x\left(\frac{1}{2} - \log \cos x\right) + C$ g

(d) None of these

7.7 DIFINITE INTEGRAL AS THE LIMIT OF SUM

80. Evaluate $\int_1^2 x^2 dx$ as limit of sums. NCERT Page-332/N-267

(a) 1

(b) $\frac{7}{3}$

(c) $\frac{1}{3}$

(d) 0

7.8 FUNDAMENTAL THEOREM OF CALCULUS

81. Value of $\int_0^4 \frac{1}{\sqrt{x^2+2x+3}} dx$ is NCERT Page-334/N-267

(a) $\log\left(\frac{1+\sqrt{3}}{5+3\sqrt{3}}\right)$

(b) $\log\left(\frac{5-3\sqrt{3}}{1-\sqrt{3}}\right)$
(c) $\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$
(d) None of these

82. $A_n=\int_0^{\pi/2}\frac{\sin(2n-1)x}{\sin x}dx; B_n=\int_0^{\pi/2}\left(\frac{\sin nx}{\sin x}\right)^2 dx;$
For $n\in\mathbf{N}$, then NCERT Page-335/N-268
(a) $A_{n+1}=A_n, B_{n+1}-B_n=A_{n+1}$
(b) $B_{n+1}=B_n$
(c) $A_{n+1}-A_n=B_{n+1}$
(d) None of these

83. Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and $g(x)$ be a function that satisfies $f(x)+g(x)=x^2$.
Then the value of the integral $\int_0^1 f(x)g(x)dx$, is NCERT Page-334/N-267
(a) $e+\frac{e^2}{2}+\frac{5}{2}$
(b) $e-\frac{e^2}{2}-\frac{5}{2}$
(c) $e+\frac{e^2}{2}-\frac{3}{2}$
(d) $e-\frac{e^2}{2}-\frac{3}{2}$

84. Let $f(x)$ be defined by $f(x)=\int_1^x t(t^2-3t+2)dt, 1\leq x\leq 3$, then range of $f(x)$ is
(a) $[0,4]$ NCERT Page-335/N-269
(b) $\left[-\frac{1}{4},4\right]$
(c) $\left[-\frac{1}{4},2\right]$
(d) None of these

85. If $\int_0^{\pi/4}\sec^2\theta\sin\theta d\theta=\int_0^k\frac{dx}{(\sqrt{x}+\sqrt{x+k})}$, then $k=$ NCERT Page-334/N-267
(a) $\frac{9}{16}$
(b) $\frac{9}{25}$
(c) $\frac{16}{25}$
(d) $\frac{25}{16}$

86. $\int_{1/4}^{1/2}\frac{dx}{\sqrt{x-x^2}}$ is equal to :
(a) $\frac{\pi}{6}$ NCERT Page-335/N-269
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$
(d) 0

87. $\int_1^2 e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx=$ NCERT Page-336/N-269
(a) $e\left(\frac{e}{2}-1\right)$
(b) $e(e-1)$

(c) 0
(d) None of these

88. $\int_0^1 x^2 e^x dx =$ NCERT Page-336/N-269
(a) e-2
(b) $e + 2$
(c) $e^2 - 2$
(d) e^2

89. The value of the integral NCERT Page-335/N-268
$\int_1^3 \left(\tan^{-1} \frac{x}{x^2+1} + \tan^{-1} \frac{x^2+1}{x}\right) dx$ is equal to
(a) π
(b) 2π
(c), 4π
(d) None

90. If $a_n = \int_0^{\frac{\pi}{2}} \frac{\sin^2 nx}{\sin x} dx$ then $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots \ldots \ldots \ldots$ are in NCERT PAGE- 335/N-269
(a) A.P.
(b) GP.
(c) H.P.
(d) None of these

91. $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ is NCERT Page-334/N-270
(a) $\frac{\pi ab}{a+b}$
(b) $\frac{\pi}{2(a+b)}$
(c) $\frac{\pi}{2ab(a+b)}$
(d) $\frac{\pi(a+b)}{2ab}$

7.9 EVALUATION OF DEFINITE INTEGRALS BY SUBSTITUTION

92. $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) dx =$ NCERT Page-338/N-272
(a) $\frac{\pi}{\sqrt{2}}$
(b) $\pi\sqrt{2}$
(c) $\frac{\pi}{2}$
(d) $\frac{\sqrt{2}}{\pi}$

93. If $\int_{\log 2}^{x} \frac{du}{(e^u-1)^{1/2}} = \frac{\pi}{6}$, then $e^x =$
(a) 1 NCERT Page-337/N-272
(b) 2
(c) 4
(d) -1

94. The correct value of $\int_0^{\pi/2} \sin x \sin 2x$ is NCERT Page-338/N-271

(a) $\frac{4}{3}$

(b) $\frac{1}{3}$

(c) $\frac{3}{4}$

(d) $\frac{2}{3}$

95. $\int_0^{\pi/2} \sqrt{\cos \theta} \sin^3 \theta d\theta =$ NCERT Page-337/N-272

(a) $\frac{20}{21}$

(b) $\frac{8}{21}$

(c) $\frac{-20}{21}$

(d) $\frac{-8}{21}$

96. If $f(\alpha) = \int_1^{\alpha} \frac{\log_{10} t}{1+t} dt, \alpha > 0$, then $f(e^3) + f(e^{-3})$ is equal to: NCERT Page-336/N-272

(a) 9

(b) $\frac{9}{2}$

(c) $\frac{9}{\log_e (10)}$

(d) $\frac{9}{2\log_e (10)}$

97. The integral $\int_0^{\frac{\pi}{2}} \frac{1}{3+2\sin x+\cos x} dx$ is equal to: NCERT Page-345/N-273

(a) $\tan^{-1} (2)$

(b) $\tan^{-1} (2) - \frac{\pi}{4}$

(c) $\frac{1}{2} \tan^{-1} (2) - \frac{\pi}{8}$

(d) $\frac{1}{2}$

98. If $[t]$ denotes the greatest integer $\leq t$, then the value of

$\int_0^1 [2x - |3x^2 - 5x + 2| + 1] dx$ is: NCERT Page-351/N-272

(a) $\frac{\sqrt{37}+\sqrt{13}-4}{6}$

(b) $\frac{\sqrt{37}-\sqrt{13}-4}{6}$

(c) $\frac{-\sqrt{37}-\sqrt{13}+4}{6}$

(d) $\frac{-\sqrt{37}+\sqrt{13}+4}{6}$

99. The minimum value of the twice differentiable function $f(x) = \int_0^x e^{x-t} f'(t) dt - (x^2 - x + 1)e^x, x \in R$, is

(a) $-\frac{2}{\sqrt{e}}$ NCERT Page-340/N-272

(b) $-2\sqrt{e}$

(c) $-\sqrt{e}$

(d) $\frac{2}{\sqrt{e}}$

100. Let $I_n(x)=\int_0^x \frac{1}{(t^2+5)^n}dt, n=1,2,3,\ldots$ Then: NCERT Page-340/N-271

(a) $50I_6-9\mathrm{I}_5=x\mathrm{I}_5$
(b) $50I_6-11I_5=xI'$
(c) $50I_6-9\mathrm{I}_5=\mathrm{I}_5$
(d) $50I_6-11\mathrm{I}_5=\mathrm{I}_5$

101. $I=\int_{\pi/4}^{\pi/3}\left(\frac{8\sin x-\sin 2x}{x}\right)dx$. Then

NCERT Page-340/N-271

(a) $\frac{\pi}{2}<I<\frac{3\pi}{4}$
(b) $\frac{\pi}{5}<I<\frac{5\pi}{12}$
(c) $\frac{5\pi}{12}<I<\frac{\sqrt{2}}{3}\pi$
(d) $\frac{3\pi}{4}<I<\pi$

102. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined as
$f(x)=a\sin\left(\frac{\pi[x]}{2}\right)+[2-x], a\in\mathbb{R}$, where $[t]$ is the greatest integer less than or equal to t. If $\lim_{x\to-1}f(x)$ exists, then the value of $\int_0^4 f(x)dx$ is equal to: NCERT Page-351/N-271

(a) -1
(b) -2
(c) 1
(d) 2

103. Evaluate: $\int_0^{\pi/2}\frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx$

NCERT Page-339/N-271

(a) $2-\sqrt{2}$
(b) $2+\sqrt{2}$
(c) $3+\sqrt{3}$
(d) $3-\sqrt{3}$

104. $\int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$ is equal to: NCERT Page-340/N-272

(a) $\sqrt{3}$
(b) $\frac{1}{\sqrt{3}}$
(c) $\frac{3\sqrt{3}}{2}$
(d) $\frac{1}{2\sqrt{3}}$

105. Evaluate: $\int_0^{\pi}\frac{1}{5+4\cos x}dx$ NCERT Page-338/N-271

(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{6}$

106. Let $F(x)=f(x)+f\left(\frac{1}{x}\right)$, where $f(x)=\int_l^x\frac{\log t}{1+t}dt$, Then $F(e)$ equals NCERT Page-338/N-271

(a) 1
(b) 2

(c) 1/2
(d) 0
107. $\int_1^{37} \frac{\pi \sin(\pi \log_e x)}{x} dx$ is equal to NCERT Page-337/N-271
(a) 2
(b) 20
(c) $\frac{2}{\pi}$
(c) 2π
108. $\int_0^1 \tan^{-1} x dx =$
(a) $\frac{\pi}{4} - \frac{1}{2}\log 2$
(b) $\pi - \frac{1}{2}\log 2$
(c) $\frac{\pi}{4} - \log 2$
(d) $\pi - \log 2$

7.10 SOME PROPERTIES OF DEFINITE INTEGRALS

109. The value of the integral $\int_{-2}^{2} \frac{|x^3+x|}{(e^{x|x|}+1)} dx$ is equal to:
(a) $5e^2$
(b) $3e^{-2}$
(c) $\frac{\pi}{4}$
(d) 6
110. Let the slope of the tangent to a curve $y = f(x)$ at $(x; y)$ be given by $2\tan x(\cos x - y)$. If the curve passes through the point, $\left(\frac{\pi}{4}, 0\right)^{9.5}$, then the value of $\int_0^{\pi/2} y dx$ is equal to
(a) $(2-\sqrt{2}) + \frac{\pi}{\sqrt{2}}$
(b) $2 - \frac{\pi}{\sqrt{2}}$
(c) $(2+\sqrt{2}) + \frac{\pi}{\sqrt{2}}$
(d) $2 + \frac{\pi}{\sqrt{2}}$
111. Let $f: \mathbf{R} \to \mathbf{R}$ be continuous function satisfying $f(x) + f(x+k) = n$, for all $x \in \mathbf{R}$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x)dx$ and $I_2 = \int_{-k}^{3k} f(x)dx$, then
(a) $I_1 + 2I_2 = 4nk$ NCERT Page-351/N-275
(c) $I_1 + nI_2 = 4n^2k$
(b) $I_1 + 2I_2 = 2nk$
(d) $I_1 + nI_2 = 6n^2k$
112. Let $f: \mathbf{R} \to \mathbf{R}$ be a differentiable function such that $f\left(\frac{\pi}{4}\right) = \sqrt{2}, f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$ and let $g(x) = \int^{\pi/4} (f'(t)\sec t + \tan t \sec t f(t))dt$ for $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim_{x \to \left(\frac{\pi}{2}\right)^{-1}} g(x)$ is equal to
(a) 2
(b) 3

(c) 4
(d) -3

113. Let $[t]$ denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 [-8x^2 + 6x - 1]dx$ is equal to NCERT Page-345/N-274

(a) -1
(b) $-\frac{5}{4}$
(c) $\frac{\sqrt{17}-13}{8}$
(d) $\frac{\sqrt{17}-16}{8}$

114. If $\int_0^2 (\sqrt{2x} - \sqrt{2x - x^2})dx = \int_0^1 \left(1 - \sqrt{1 - y^2} - \frac{y^2}{2}\right) dy + \int_1^2 \left(2 - \frac{y^2}{2}\right) dy + I$ NCERT Page-346/N-277

(a) $\int_0^1 (1 + \sqrt{1 - y^2})dy$
(b) $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1 - y^2} + 1\right) dy$
(c) $\int_0^1 (1 - \sqrt{1 - y^2})dy$
(d) $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1 - y^2} + 1\right) dy$

EXERCISE 2 : NCERT EXEMPLAR & PAST YEARS JEE MAIN

NCERT EXEMPLAR QUESTIONS

1. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to NCERT Page-296/N-226

(a) $2(\sin x + x\cos \theta) + C$
(b) $2(\sin x - x\cos \theta) + C$
(c) $2(\sin x + 2x\cos \theta) + C$
(d) $2(\sin x - 2x\cos \theta) + C$

2. $\frac{dx}{\sin (x-a)\sin (x-b)}$ is equal to NCERT Page-295/N-226

(a) $\sin (b - a)\log \left|\frac{\sin (x-b)}{\sin (x-a)}\right| + C$
(b) $\operatorname{cosec} (b - a)\log \left|\frac{\sin (x-a)}{\sin (x-b)}\right| + C$
(c) $\operatorname{cosec} (b - a)\log \left|\frac{\sin (x-b)}{\sin (x-a)}\right| + C$
(d) $\sin (b - a)\log \left|\frac{\sin (x-a)}{\sin (x-b)}\right| + C$

3. $\int \frac{x^9}{(4x^2+1)^6} dx$ is equal to NCERT Page-295/N-239

(a) $\frac{1}{5x}\left(4 + \frac{1}{x^2}\right)^{-5} + C$
(b) $\frac{1}{5}\left(4 + \frac{1}{x^2}\right)^{-5} + C$

(c) $\frac{1}{10x}\left(\frac{1}{x}+4\right)^{-5}+\mathrm{C}$
(d) $\frac{1}{10}\left(\frac{1}{x^2}+4\right)^{-5}+\mathrm{C}$

4. If $\int \frac{dx}{(x+2)(x^2+1)} = a\log|1+x^2| + b\tan^{-1} x + \frac{1}{5}\log|x+2| + C$, then

NCERT Page-297/N-256

(a) $a = \frac{-1}{10}, b = \frac{-2}{5}$
(b) $a = \frac{1}{10}, b = -\frac{2}{5}$
(c) $a = \frac{-1}{10}, b = \frac{2}{5}$
(d) $a = \frac{1}{10}, b = \frac{2}{5}$

5. $\int \frac{x^3}{x+1}$ is equal to

NCERT Page-298/N-226

(a) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + C$
(b) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + C$
(c) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + C$
(d) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + C$

6. $\int \frac{x+\sin x}{1+\cos x}$ is equal to

NCERT Page-300/N-236

(a) $\log|1+\cos x| + C$
(b) $\log|x+\sin x| + C$
(c) $x - \tan\frac{x}{2} + C$
(d) $x \cdot \tan\frac{x}{2} + C$

7. If $\frac{x^3 dx}{\sqrt{1+x^2}} = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$, then.

NCERT Page-314/N-256

(a) $a = \frac{1}{3}, b = 1$
(b) $a = \frac{-1}{3}, b = 1$
(c) $a = \frac{-1}{3}, b = -1$
(d) $a = \frac{1}{3}, b = -1$

8. $\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ is equal to

NCERT Page-325/N-277

(a) 1
(b) 2
(c) 3
(d) 4

9. $\int_0^{\pi/2} \sqrt{1-\sin 2x}$ is equal to

NCERT Page-326/N-229

(a) $2\sqrt{2}$
(b) $2(\sqrt{2}+1)$
(c) 2
(d) $2(\sqrt{2}-1)$

10. $\int_0^{\pi/4} \cos x e^{\sin x} dx$ is equal to NCERT Page-327/N-236

(a) $e+1$

(b) $e-1$

(c) e

(d) $-e$

11. $\int \frac{x+3}{(x+4)^2} e^x dx$ is equal to NCERT Page-326/N-236

(a) $e^x\left(\frac{1}{x+4}\right)+C$

(b) $e^{-x}\left(\frac{1}{x+4}\right)+C$

(c) $e^{-x}\left(\frac{1}{x-4}\right)+C$

(d) $e^{2x}\left(\frac{1}{x-4}\right)+C$

PAST YEARS JEE MAIN

12. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3} dx$ is equal to : NCERT Page-303/N-272 | 2016, A

(a) $\frac{x^5}{2(x^5+x^3+1)^2}+C$

(b) $\frac{-x^{10}}{2(x^5+x^3+1)^2}+C$

(c) $\frac{-x^5}{(x^5+x^3+1)^2}+C$

(d) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$

where C is an arbitrary constant.

13. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\mathrm{dx}}{1+\cos \mathrm{x}}$ is equal to :

(a) -1

(b) -2

(c) 2

(d) $4^{2017,A}$

14. Let $I_n=\int \tan^n x dx, (n>1). I_4+I_6=a\tan^5 x+bx^5+C$, where C is constant of integration, then the ordered pair (a,b) is equal to : NCERT Page-303/N-260 | 2017, A

(a) $\left(-\frac{1}{5},0\right)$

(b) $\left(-\frac{1}{5},1\right)$

(c) $\left(\frac{1}{5},0\right)$

(d) $\left(\frac{1}{5},-1\right)$

15. The integral NCERT Page-304/N-237 | 2018, S

$\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2} dx$ is equal to:

(a) $\frac{-1}{3(1+\tan^3 x)}+C$

(b) $\frac{1}{1+\cot^3 x}+C$

(c) $\frac{-1}{1+\cot^3 x}+C$
(c) $\frac{1}{3(1+\tan^3 x)}+C$
(where C is a constant of integration)
16. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is :
(a) $\frac{\pi}{2}$ NCERT Page-346/N-274 | 2018, A
(b) 4π
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{8}$
17. The value of $\int_0^{\pi} |\cos x|^3 \mathrm{d}x$ is: NCERT Page-341/N-284 | 2019, C
(a) 0
(b) $\frac{4}{3}$
(c) $\frac{2}{3}$
(d) $\frac{-4}{3}$
18. For $x^2 \neq \mathrm{n}\pi + 1, \mathrm{n} \in \mathrm{N}$ (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx$
is equal to: NCERT Page-305/N-282 | 2019, C
(a) $\log_e \left|\frac{1}{2}\sec^2(x^2-1)\right| + \mathrm{c}$
(b) $\frac{1}{2}\log_e |\sec(x^2-1)| + \mathrm{c}$
(c) $\frac{1}{2}\log_e \left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$
(d) $\log_e \left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$
(where c is a constant of integration)
19. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to: NCERT Page-304/N-281 | 2019, C
(a) $-3\tan^{-1/3} x + C$
(c) $-\frac{3}{4}\tan^{-4/3} x + C$
(c) $-3\cot^{-1/3} x + C$
(d) $3\tan^{-1/3} x + C$
(Here C is a constant of integration)
20. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x+\cos x} dx$ is: NCERT Page-342/N-274 | 2019, C
(a) $\frac{\pi-2}{8}$
(b) $\frac{\pi-1}{4}$
(c) $\frac{\pi-2}{4}$
(d) $\frac{\pi-1}{2}$
21. If $\int \frac{\cos x dx}{\sin^3 x(1+\sin^6 x)^{2/3}} = f(x)(1+\sin^6 x)^{1/\lambda} + c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to:

(a) $-\frac{9}{8}$
(b) 2
(c) $\frac{9}{8}$
(d) -2

22. If $\int \frac{\sin x}{\sin^3 x+\cos^3 x} dx$
$=\alpha \log_e |1+\tan x|+\beta \log_e |1-\tan x+\tan^2 x|+\gamma \tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$, when C is constant of integration, then the value of $18(\alpha+\beta+\gamma^2)$ is : NCERT Page-325/N-236 | 2021, C

23. If $[x]$ is the greatest integer $\leq x$, then $\pi^2 \int_0^2 \left(\sin \frac{\pi x}{2}\right)(x-[x])^{[x]} dx$ is equal to :
(a) $2(\pi-1)(b) 4(\pi-1)$ NCERT Page-333/N-276 | 2021, S
(c) $4(\pi+1)$
(d) $2(\pi+1)$

24. If $a=\lim_{n \to \infty} \sum_{k=1}^{n} \frac{2n}{n^2+k^2}$ and $f(x)=\sqrt{\frac{1-\cos x}{1+\cos x}}, x \in (0,1)$, then:
(a) $2\sqrt{2} f\left(\frac{a}{2}\right)=f'\left(\frac{a}{2}\right)$ NCETR Page-333/N-276 | 2022, C
(b) $f\left(\frac{a}{2}\right) f'\left(\frac{a}{2}\right)=\sqrt{2}$
(c) $\sqrt{2} f\left(\frac{a}{2}\right)=f'\left(\frac{a}{2}\right)$
(d) $f\left(\frac{a}{2}\right)=\sqrt{2} f'\left(\frac{a}{2}\right)$

25. If $n(2n+1)\int_0^1 (1-x^n)^{2n} dx=1177\int_0^1 (1-x^n)^{2n+1} dx$,
then $n \in N$ is equal to NCERT Page-336/N-278 | 2022, S

26. If $I(x)=\int e^{\sin^2 x}(\cos x \sin 2x-\sin x) dx$ and $I(0)=1$, then $I\left(\frac{\pi}{3}\right)$ is equal to
(a) $-\frac{1}{2} e^{\frac{3}{4}}$
(b) $e^{\frac{3}{4}}$ NCERT Page-324/N-261 | 2023, C
(c) $\frac{1}{2} e^{\frac{3}{4}}$
(d) $-e^{\frac{3}{4}}$

27. $\int_0^{\infty} \frac{6}{e^{3x}+6e^{2x}+11e^x+6} dx$
(a) $\log_e\left(\frac{512}{81}\right)$ NCERT Page-338/N-271 | 2023, S
(b) $\log_e\left(\frac{32}{27}\right)$
(c) $\log_e\left(\frac{256}{81}\right)$
(d) $\log_e\left(\frac{64}{27}\right)$

EXERCISE 3 : SKILL ENHANCER MCQS

1. $\int (x+3)\sqrt{3-4x-x^3}dx$ is equal to
   (a) $-\frac{1}{3}(3-4x-x^3)^{3/2}+\frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right)+\frac{(x+2)\sqrt{3-4x-x^2}}{2}+C$
   (b) $\frac{1}{3}(3-4x-x^3)^{3/2}+\frac{7}{2}\sin^{-1}\left(\frac{x+2}{7}\right)+\frac{(x+2)\frac{2}{\sqrt{3-4x-x^2}}}{2}+C$
   (c) $\frac{1}{3}(3-4x-x^3)^{3/2}+\frac{7}{2}\sin^{-1}\left(\frac{x+7}{2}\right)+\frac{(x+7)\sqrt{3-4x-x^2}}{2}+C$
   (d) $\frac{1}{3}(3-4x-x^3)^{3/2}+\frac{7}{2}\log\left[x+7+\frac{2}{\sqrt{3-4x-x^2}}\right]+C$
2. For $0<x<1$, let $f(x)=\lim_{n\to\infty}(1+x)(1+x^2)(1+x^4)\ldots..(1+x^{2^n})$ then $\int \frac{f(x)}{1-x}\log_e xdx$ equals
   (a) $\log_e\left(\frac{x}{1-x}\right)+c$
   (b) $-\log_e\left(\frac{x}{1-x}\right)+\frac{\log_e x}{1-x}+c$
   (c) $\frac{\log_e x}{1-x}+\log_e(1-x)+c$
   (d) $x\log_e x+\log_e(1-x)+c$
3. $\int \frac{x^2-1}{x\sqrt{(x^2+\alpha x+1)(x^2+\beta x+1)}}dx=$
   (a) $2\log\left\{\frac{\sqrt{x^2+\alpha x+1}+\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\}+c$
   (b) $2\log\left\{\frac{\sqrt{x^2+\alpha x+1}-\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\}+c$
   (c) $\log\left\{\sqrt{x^2+\alpha x+1}-\sqrt{x^2+\beta x-1}\right\}+c$
   (d) None of these
4. If $\int \frac{x}{x^8+1}\mathrm{dx}=\frac{1}{k_1}\tan^{-1}\left(\frac{x^4-1}{k_2x^2}\right)-\frac{1}{2}\int\frac{x^5-x}{x^8+1}+C$, then value of k_1k_2 is
   (where C is an integration constant)
   (a) 8
   (b) 2
   (c) $4\sqrt{2}$
   (d) 4
5. If $\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}}=-[f(x)]^{1/n}+C$, then f(x) is
   (a) $(1+x^n)$
   (b) $1+x^{-n}$
   (c) x^n+x^{-n}
   (d) None of these
6. The value of $\int_a^b \frac{dx}{1+x^{2n+1}+\sqrt{1+x^{4n+2}}}$ where $n\in N$ and a and b are local maximum and minimum point of $g(x)=\int_0^x (t+2016)^7(t+2014)^{110}(t-2016)^5\mathrm{dt}$ respectively, is
   (a) 0
   (b) 2016
   (c) 2014
   (d) $2n+1$

7. If $\alpha, \beta (\beta > \alpha)$ are the roots of $f(x) \equiv ax^2 + bx + c = 0, a \neq 0$ and $f(x)$ is an even function and $I = \int_{\alpha}^{\beta} \frac{e^{f\left(\frac{f(x)}{x-\alpha}\right)}}{e^{f\left(\frac{f(x)}{x-\alpha}\right)} + e^{f\left(\frac{f(x)}{x-\beta}\right)}}$, then $|\eta|$ is equal to
   (a) $\left|\frac{b}{a}\right|$
   (b) $\left|\frac{b}{2a}\right|$
   (c) $\frac{\sqrt{b^2-4ac}}{|2a|}$
   (d) None of these
8. $\text{f}: [0,1] \to \text{R}$ is a function with continuous second derivative and $\text{f}'(\text{x}) \in (0,1] \forall \text{x} \in [0,1]$.
   Let $\int_0^x f(t)dt = g(x), f(0) = g(0)$ and
   $\int_0^x (f(t))^3 dt = h(x)$, then
   (a) $\text{h}(\text{x}) \leq \text{g}(\text{x})$
   (c) $2\text{g}(\text{x}) \geq (\text{f}(\text{x}))^2$
   (b) $(g(x))^2 \geq h(x)$
   (d) $\text{f}(1) = 1$
9. If p, q, r, s are in arithmetic progression and $f(x) = \begin{vmatrix} p + \sin x & q + \sin x & p - r + \sin x \\ q + \sin x & r + \sin x & -1 + \sin x \\ r + \sin x & s + \sin x & s - q + \sin x \end{vmatrix}$ such that
   $\int_0^2 f(x)dx = -4$, then the common difference of the progession is
   (a) ± 1
   (b) $\frac{1}{2}$
   (c) ± 2
   (d) None of these
10. Let $g(x) = \int_0^x f(t)dt$, where f is such that $\frac{1}{2} \leq f(t) \leq 1$, for $t \in [0,1]$ and $0 \leq f(t) \leq \frac{1}{2}$, for $t \in [1,2]$. Then $g(2)$ satisfies the inequality
    (a) $-\frac{3}{2} \leq g(2) < \frac{1}{2}$
    (b) $0 \leq g(2) < 2$
    (c) $\frac{3}{2} < g(2) \leq \frac{5}{2}$
    (d) $2 < g(2) < 4$
11. Let $f: R \to R$ be a differentiable function and $f(1) = 4$. Then the value of $\lim_{x \to 1} \frac{\int_4^{f(x)} 2t dt}{x-1}$, if $f'(1) = 2$ is
    (a) 16
    (b) 8
    (c) 4
    (d) 2
12. If $\lim_{x \to 0} \frac{f(x)}{x^2}$ exists finitely and $\lim_{x \to 0} \left(1 + x + \frac{f(x)}{x}\right)^{1/x} = e^3$, then $\int f(x) \log_e x dx$ is equal to
    (a) $\frac{2}{3}x^3 \left(\log_e x - \frac{1}{3}\right) + c$
    (b) $\frac{x^3}{3} \left(\log_e x - \frac{1}{3}\right) + c$

(c) $\frac{2}{3}x^3(\log_e\ x+1)+c$
(d) $\frac{2}{3}x^3(\log_e\ x-1)+c$

13. If $f:\mathbf{R}\to\mathbf{R}$ is a function satisfying the following:
(i) $f(-x)=-f(x)$
(iii) $f\left(\frac{1}{x}\right)=\frac{f(x)}{x^2}\forall x\neq 0$ then $\int e^x f(x)dx$ is equal to
(a) $e^x(x-1)+c$
(b) $e^x\log\ x+c$
(c) $\frac{e^x}{x}+c$
(d) $\frac{e^x}{x+1}+c$
(ii) $f(x+1)=f(x)+1$

14. If $\int\frac{\text{cosec}^2\ x-2010}{\cos^{2010}\ x}dx=-\frac{f(x)}{(g(x))^{2010}}+C$; where $f\left(\frac{\pi}{4}\right)=1$; then the number of solutions of the equation $\frac{f(x)}{g(x)}=\{x\}$ in $[0,2\pi]$ is/are : (where {. represents fractional part function)
(a) 0
(b) 1
(c) 2
(d) 3

15. $\int\frac{1}{x}\{\log_{ex}\ e\cdot\log_{e^2x}\ e\cdot\log_{e^3x}\ e\}dx$ is equal to
(a) $\frac{1}{2}\log\{\log_e\ e^2x\}-\log\{\log_e\ e^3x\}+\log\{\log_e\ e^4x\}+C$
(b) $\frac{1}{2}\log\{\log_e\ x\}-\log\{\log_e\ x\}+\log\{\log_e\ x\}+C$
(c) $\frac{1}{2}\log\{\log_e\ ex\}-\log\{\log_e\ e^2x^2\}+\log\{\log_e\ e^3x^3\}+C$
(d) $\frac{1}{2}\log\{\log_e\ ex\}-\log\{\log_e\ e^2x\}+\frac{1}{2}\log\{\log_e\ e^3x\}+C$

EXERCISE 4: NUMERIC VALUE ANSWER QUESTIONS

1. If $\int_0^{\sqrt{3}}\frac{15x^3}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}dx=\alpha\sqrt{2}+\beta\sqrt{3}$, where α,β are integers, then $\alpha+\beta$ is equal to
2. The value of the integral $\int_0^{\frac{\pi}{2}}60\frac{\sin(6x)}{\sin x}dx$
3. Let $f(x)=\min\{[x-1],[x-2],\ldots,[x-10]\}$ where $[t]$ denotes the greatest integer $\leq t$. Then is equal to
$\int_0^{10}f(x)dx+\int_0^{10}(f(x))^2dx+\int_0^{10}|f(x)|dx$
4. Let f be a differentiable function satisfying $f(x)=\frac{2}{\sqrt{3}}\int_0^{\sqrt{3}}f\left(\frac{\lambda^2x}{3}\right)d\lambda, x>0$ and $f(1)=\sqrt{3}$ If $y=f(x)$ passes through the point $(\alpha,6)$, then α is equal to
5. Let f be a twice differentiable function on R. If $f'(0)=4$ and $f(x)+\int_0^x(x-t)f'(t)dt=(e^{2x}+e^{-2x})\cos\ 2x+\frac{2}{a}x$, then $(2a+1)^5a^2$ is equal to
6. Let $a_n=\int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\cdots\ldots.+\frac{x^{n-1}}{n}\right)dx$ for $n\in N$. Then the sum of all the elements of the set $\{n\in N: a_n\in(2,30)\}$ is

7. If $\int_0^x f(\mathrm{t})\mathrm{dt} = x^2 + \int_x^1 \mathrm{t}^2 f(\mathrm{t})\mathrm{dt}$, if $f'(1/2)$ is $\frac{24}{5k}$, then the value of k is
8. $I_n = \int_0^{\pi/4} \tan^n xdx$ then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ is equal to
9. Let f be a positive function.
   Let $I_1 = \int_{1-k}^{k} xf[\dot{x}(1-x)]dx$,
   $I_2 = \int_{1-k}^{k} f[x(1-x)]dx$, where $2k-1>0$. Then $\frac{I_1}{I_2}$ is
10. Let $f: R \to R$ and g: $R \to R$ be continuous functions. Then the value of the integral
    $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x)+f(-x)][g(x)-g(-x)]dx$ is
11. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$, where C is a constant of integration. If the function $f(x)$ is equal to $-\frac{C}{2x^3}$. Then, the value of C is
12. The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)dx$ is equal to $\frac{\pi}{4} - \frac{k}{2}\ln\ 2$, (natural log $\ln$) then k is

Answer Keys

EXCERSISE- 1

1	(a)	13	(d)	25	(a)	37	(b)	49	(b)	61	(a)	73	(a)	85	(a)	97	(b)	109	(d)
2	(c)	14	(b)	26	(a)	38	(a)	50	(b)	62	(d)	74	(b)	86	(a)	98	(a)	110	(b)
3	(a)	15	(a)	27	(b)	39	(b)	51	(b)	63	(a)	75	(b)	87	(a)	99	(a)	111	(c)
4	(c)	16	(a)	28	(d)	40	(c)	52	(a)	64	(d)	76	(c)	88	(a)	100	(a)	112	(b)
5	(b)	17	(a)	29	(b)	41	(c)	53	(b)	65	(b)	77	(c)	89	(a)	101	(c)	113	(c)
6	(a)	18	(c)	30	(b)	42	(c)	54	(a)	66	(d)	78	(b)	90	(c)	102	(b)	114	(c)
7	(a)	19	(d)	31	(b)	43	(a)	55	(c)	67	(a)	79	(c)	91	(c)	103	(a)		
8	(a)	20	(b)	32	(b)	44	(c)	56	(b)	68	(b)	80	(b)	92	(b)	104	(a)		
9	(a)	21	(a)	33	(b)	45	(b)	57	(a)	69	(c)	81	(c)	93	(c)	105	(a)		
10	(b)	22	(d)	34	(b)	46	(a)	58	(b)	70	(b)	82	(a)	94	(d)	106	(c)		
11	(b)	23	(b)	35	(c)	47	(b)	59	(d)	71	(b)	83	(d)	95	(b)	107	(a)		
12	(c)	24	(b)	36	(a)	48	(a)	60	(c)	72	(b)	84	(c)	96	(d)	108	(a)		

EXCERSISE- 2

1	(a)	4	(c)	7	(d)	10	(b)	13	(c)	16	(c)	19	(a)	22	(3)	25	(24)		
2	(c)	5	(d)	8	(a)	11	(a)	14	(c)	17	(b)	20	(b)	23	(b)	26	(c)		
3	(d)	6	(d)	9	(d)	12	(d)	15	(a)	18	(c,d)	21	(d)	24	(c)	27	(b)		

EXCERSISE- 3																			
1	(a)	3	(a)	5	(b)	7	(c)	9	(a)	11	(a)	13	(a)	15	(d)				
2	(b)	4	(a)	6	(b)	8	(b)	10	(b)	12	(a)	14	(a)						
EXCERSISE- 4																			
1	(10)	3	(385)	5	(8)	7	(2)	9	(0.5)	11	(1)								
2	(104)	4	(12)	6	(5)	8	(1)	10	(0)	12	(1)								

HINTS AND SOLUTIONS

EXERCISE - 1

1. (a) $\int x^{51}(\tan^{-1} x+\cot^{-1} x)dx$
$= \int x^{51}\cdot\frac{\pi}{2}dx \quad \left\{\because \tan^{-1} x+\cot^{-1} x=\frac{\pi}{2}\right\}$
$= \frac{\pi x^{52}}{104}+c=\frac{x^{52}}{52}(\tan^{-1} x+\cot^{-1} x)+c.$

2. (c) $\int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx=\int \frac{x^5-x^4}{x^3-x^2}dx$
$= \int \frac{x^4(x-1)}{x^2(x-1)}dx=\int x^2dx=\frac{x^3}{3}+c.$

3. (a) $\int (x^4+x^2+2x^3)dn=\frac{x^5}{5}+\frac{x^3}{3}+\frac{x^4}{2}+c.$

4. (c) $\int \frac{(x-3)(x-2)}{x-2}dx=\int (x-3)dx=\frac{x^2}{2}-3x+c$

5. (b) $\int \left(x^{2-\frac{1}{2}}+x^{1-\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx=\int \left(x^{\frac{3}{2}}+x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx=\frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}+2x^{1/2}+c$

6. (a) Consider $\int \frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$

$$=\int \frac{\sin^2 xdx}{\sin^2 x\cos^2 x}-\int \frac{\cos^2 xdx}{\sin^2 x\cos^2 x}$$
$$=\int \sec^2 xdx-\int \operatorname{cosec}^2 xdx$$
Let $I=\int (\sec^2 x-\operatorname{cosec}^2 x)dx$
$$=\tan x-(-\cot x)+c=\tan x+\cot x+c$$

7. (a) Given integral is

$$I(x)=\int \sec^2 x\cdot \sin^{-2022} x dx-2022\int \sin^{-2022} x dx$$

$$=\tan x\cdot(\sin x)^{-2022}+\int^{I}((2022)\tan x\cdot(\sin x)^{-2023}x dx$$

$$-2022\int (\sin x)^{-2022} dx$$

$$\text{At } x=\pi/4, 2^{1011}=\left(\frac{1}{\sqrt{2}}\right)^{-2022}+C$$

$$\text{so, } C=0$$

Hence $I(x)=\frac{\tan x}{(\sin x)^{2022}}$

Put $x=\frac{\pi}{6}$ in the above expression,

$$I\left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}\left(\frac{1}{2}\right)^{2022}}=\frac{2^{2022}}{\sqrt{3}}$$

$$I\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}}=\frac{2^{2022}}{(\sqrt{3})^{2021}}=\frac{1}{3^{1010}}I\left(\frac{\pi}{6}\right)$$

$$3^{1010}I\left(\frac{\pi}{3}\right)=I\left(\frac{\pi}{6}\right)$$

8. (a) $I=\int x^x(1+\log x)dx$

Put $x^x=t$, then $x^x(1+\log x)dx=dt$

$\therefore \quad I=\int dt \Rightarrow I=t+C \Rightarrow I=x^x+C.$

9. (a) Let $I=\int \frac{\cos 8x+1}{\cot 2x-\tan 2x}dx$

Now, $D^r=\cot 2x-\tan 2x=\frac{\cos 2x}{\sin 2x}-\frac{\sin 2x}{\cos 2x}$

$$=\frac{\cos^2 2x-\sin^2 2x}{\sin 2x\cos 2x}=\frac{2\cos 4x}{\sin 4x}$$

$$\therefore \quad I=\int \frac{2\cos^2 4x}{\frac{2\cos 4x}{\sin 4x}}dx=\int \frac{2\cos^2 4x\cdot \sin 4x}{2\cos 4x}dx$$

$$=\frac{1}{2}\int \sin 8x dx=-\frac{1}{2}\frac{\cos 8x}{8}+k=-\frac{1}{16}\cdot \cos 8x+k$$

Now, $-\frac{1}{16}\cdot \cos 8x+k=A\cos 8x+k \Rightarrow A=-\frac{1}{16}$

10. (b) Put $x^{3/2}=t \Rightarrow \frac{3}{2}x^{1/2}dx=dt$

$\therefore$ integral is

$$\int \frac{\frac{2}{3}dt}{\sqrt{1-t^2}}=\frac{2}{3}\sin^{-1} t+C=\frac{2}{3}\sin^{-1}\left(x^{3/2}\right)+C$$

11. (b) $\int \sec^{2/3} x\operatorname{cosec}^{4/3} x dx=\int \frac{dx}{\sin^{4/3} x\cos^{2/3} x}$

Multiplying N^r and D^r by $\cos^2 x$, we get

$\{$ Putting $\tan x=t \Rightarrow \sec^2 x dx=dt\}$

$=\int \frac{\sec^2 x dx}{\tan^{4/3} x}=\int \frac{dt}{t^{4/3}}=\frac{t^{-1/3}}{(-1/3)}+c=-3(\tan x)^{-1/3}+c.$

12.

(c) $I=\int \frac{dx}{\cos x+\sqrt{3}\sin x} \Rightarrow I=\int \frac{dx}{2\left[\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x\right]}$

$=\frac{1}{2}\int \frac{\mathrm{dx}}{\left[\sin \frac{\pi}{6}\cos \mathrm{x}+\cos \frac{\pi}{6}\sin \mathrm{x}\right]}=\frac{1}{2}\cdot\int \frac{\mathrm{dx}}{\sin\left(\mathrm{x}+\frac{\pi}{6}\right)}$

$\Rightarrow \mathrm{I}=\frac{1}{2}\cdot\int \operatorname{cosec}\left(\mathrm{x}+\frac{\pi}{6}\right)\mathrm{dx}$

$\because \int \operatorname{cosec} \mathrm{x} d\mathrm{x}=\log|(\tan \mathrm{x}/2)|+\mathrm{C}$

$\therefore \mathrm{I}=\frac{1}{2}\cdot\log \tan\left(\frac{\mathrm{x}}{2}+\frac{\pi}{12}\right)+\mathrm{C}$

13. (d) Given integral is

$\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{xg(x)}{e^x+1}+c$

On differentiating both sides w.r.t. x, we get

$\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)$

$=\frac{(e^x+1)(g(x)+xg'(x))-e^x\cdot x\cdot g(x)}{(e^x+1)^2}$

$(e^x+1)x(\cos x-\sin x)+g(x)(e^x+1-xe^x)$

$=(e^x+1)(g(x)+xg'(x))-e^x\cdot xg(x)$

$\Rightarrow g'(x)=\cos x-\sin x$

Take integral both sides.

$\Rightarrow g(x)=\sin x+\cos x+C$

Take $g(x)=0$; then $x=\frac{\bar{x}}{4}$

So, $g(x)$ is increasing in $(0,\pi/4)$

Again, differentiate w.r.t. x in equation (i),

$g''(x)=-\sin x-\cos x<0$

$\Rightarrow g'(x)$ is decreasing function

Let $r(x)=g(x)+g'(x)=2\cos x+C$

$\Rightarrow r'(x)=g'(x)+g''(x)=-2\sin x<0$

$\Rightarrow r$ is decreasing

Let $l(\mathrm{x})=g(x)-g'(x)=2\sin x+C$

Differentiate w.r.t. x

$\Rightarrow l'(x)=g'(x)-g''(x)=2\cos x>0$

Take $l'(x)=0; \cos \mathrm{x}=0; \mathrm{x}=\frac{\pi}{2}$

$\Rightarrow l$ is increasing

Therefore, $l(x)$ is increasing at $\left(0,\frac{\pi}{2}\right)$

14. (b) Given integral is

$\int\left(\frac{x^2+1}{(x+1)^2}\right)e^x\cdot dx.=\int\left(\frac{x^2-1+2}{(x+1)^2}\right)e^x dx$

$=\int\left(\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right)e^x dx=\int(f(x)+f'(x))e^x dx$

$= f(x)e^x + c$

After comparing the integral the function is

$f(x) = \dfrac{x-1}{x+1}$

Then, $f'(x) = \frac{2}{(x+1)^2}$

Differentiate again two times w.r.t. x.

$f''(x) = \dfrac{-4}{(x+1)^3} \Rightarrow f'''(x) = \dfrac{12}{(x+1)^4}$

Put $x = 1$ in above equation.

$f''(1) = \dfrac{12}{16} = \dfrac{3}{4}$

15. (a) Given integral is $I = \int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos\ x-\sin\ x)}{\left(1+\frac{2}{\sqrt{3}}\sin\ 2x\right)} dx$

$I = \dfrac{\sqrt{3}}{2}\int \dfrac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos\ x-\sin\ x)}{\left(\frac{\sqrt{3}}{2}+\sin\ 2x\right)} dx$

$I = \int \dfrac{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)(\cos\ x-\sin\ x)}{\sin\ \left(\frac{\pi}{3}\right)+\sin\ 2x} dx$

$I = \int \dfrac{\left(\frac{\sqrt{3}}{2}\cos\ x-\frac{1}{2}\cos\ x-\frac{\sqrt{3}}{2}\sin\ x+\frac{1}{2}\sin\ x\right)}{2\sin\ \left(x+\frac{\pi}{6}\right)\cos\ \left(x-\frac{\pi}{6}\right)} dx$

$I = \int \dfrac{\left(\cos\ \left(x-\frac{\pi}{6}\right)-\sin\ \left(x+\frac{\pi}{6}\right)\right)}{2\sin\ \left(x+\frac{\pi}{6}\right)\cos\ \left(x-\frac{\pi}{6}\right)} dx$

$\dfrac{1}{2}\left(\int \dfrac{dx}{\sin\ \left(x+\frac{\pi}{6}\right)} - \int \dfrac{dx}{\cos\ \left(x-\frac{\pi}{6}\right)}\right)$

$\dfrac{1}{2}\left(\int \operatorname{cosec}\ \left(x+\dfrac{\pi}{6}\right) dx - \int \sec\ \left(x-\dfrac{\pi}{6}\right) dx\right)$

$\dfrac{1}{2}\ln\left|\dfrac{\tan\ \left(\frac{x}{2}+\frac{x}{12}\right)}{\tan\ \left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|$

16. (a) $\int \frac{dx}{\{(x-1)^3(x+2)^5\}^{1/4}} = \int \frac{dx}{\left(\frac{x-1}{x+2}\right)^{3/4}\cdot(x+2)^2}$ Put $\frac{x-1}{x+2} = t \Rightarrow \frac{3}{(x+2)^2}dx = dt$

$\therefore$ Integral $= \int \frac{\mathrm{dt}}{3\mathrm{t}^{3/4}} = \frac{4}{3}\mathrm{t}^{\frac{1}{4}} + \mathrm{c} = \frac{4}{3}\left(\frac{\mathrm{x}-1}{\mathrm{x}+2}\right)^{1/4} + \mathrm{c}$

17. (c)

(a) $\int \frac{2dx}{(e^x+e^{-x})^2} = \int \frac{2e^{2x}}{(e^{2x}+1)^2}dx = -\frac{1}{(\mathrm{e}^{2\mathrm{x}}+1)} + \mathrm{c} = -\frac{\mathrm{e}^{-\mathrm{x}}}{\mathrm{e}^{\mathrm{x}}+\mathrm{e}^{-\mathrm{x}}} + \mathrm{c}$

19. (d) Let $I = \int (27e^{9x} + e^{12x})^{1/3}dx = \int \mathrm{e}^{3\mathrm{x}}(27 + \mathrm{e}^{3\mathrm{x}})^{1/3}\mathrm{dx}$

Put $27 + \mathrm{e}^{3\mathrm{x}} = \mathrm{t} \Rightarrow 3\mathrm{e}^{3\mathrm{x}}\mathrm{dx} = \mathrm{dt}$

$$\therefore I=\frac{1}{3}\int t^{1/3}dt=\frac{1}{3}\times\frac{t^{4/3}}{4/3}+C=\frac{1}{4}(27+e^{3x})^{4/3}+C$$

20.

(b) $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$

Let $xe^x=t$

$$\Rightarrow (xe^x+e^x)=\frac{dt}{dx}\Rightarrow dx=\frac{dt}{e^x(x+1)}$$

$$\therefore \quad \int \frac{e^x(1+x)}{\cos^2(e^x x)}dx=\int \frac{e^x(1+x)}{\cos^2 t}\times\frac{dt}{e^x(1+x)}$$

$$=\int \frac{1}{\cos^2 t}dt=\int \sec^2 t\,dt=\tan t+C=\tan(xe^x)+C$$

21. (a) Let $I=\int 2^{2^{2^x}}2^{2^x}2^x dx$

Let $2^{2^{2^x}}=t\Rightarrow 2^{2^{2^x}}2^{2^x}2^x(\log 2)^3dx=dt$

$$\Rightarrow I=\int \frac{1}{(\log 2)^3}dt=\frac{1}{(\log 2)^3}t+C=\frac{1}{(\log 2)^3}2^{2^{2^x}}+C$$

22. (d) Put $10^x+x^{10}=t$

$$\therefore (10^x\log_e 10+10x^9)dx=dt$$

$$\therefore \int \frac{10x^9+10^x\log_e 10}{10^x+x^{10}}dx=\int \frac{dt}{t}$$

23. (b) Put $x=\cos 2\theta$

$$\therefore \quad I=\int \cos\{2\tan^{-1}\tan\theta\}(-2\sin 2\theta)d\theta$$

$$=-\int \sin 4\theta d\theta=\frac{1}{4}\cos 4\theta+c$$

$$=\frac{1}{4}(2x^2-1)+c=\frac{1}{2}x^2+k$$

24. (b) $\int e^{3\log x}(x^4+1)^{-1}dx=\int e^{\log x^3}\frac{1}{x^4+1}dx$

$$=\int \frac{x^3}{x^4+1}dx=\frac{1}{4}\log(x^4+1)+C$$

[since $e^{\log_e x^3}=x^3$]

25. (a) $I=\int \sin^3 x\cdot\cos^5 x dx$

Put $\sin x=t\Rightarrow \cos x dx=dt$

$$I=\int \sin^3 x\cdot\cos^4 x\cdot\cos x dx=\int t^3(1-t^2)^2dt$$

$$=\int (t^3-2t^5+t^7)dt=\frac{1}{4}t^4-\frac{2}{6}t^6+\frac{1}{8}t^8+C$$

$$=\frac{1}{4}\sin^4 x-\frac{1}{3}\sin^6 x+\frac{1}{8}\sin^8 x+C$$

26. (a) $I=\int \left(x+\frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right)dx$

Put $x+\frac{1}{x}=t\Rightarrow\left(1-\frac{1}{x^2}\right)dx=dt$

$$\Rightarrow\left(\frac{x^2-1}{x^2}\right)dx=dt$$

$$\therefore I=\int t^{n+5}dt=\frac{t^{n+6}}{n+6}+c=\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$$

27. (b) $I=\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$

$$=\int \frac{(\sin^4 x-\cos^4 x)(\sin^4 x+\cos^4 x)}{1-2\sin^2 x\cos^2 x}dx$$

$(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)$

$$=\int \frac{(\sin^4 x+\cos^4 x)}{1-2\sin^2 x\cos^2 x}dx$$

1. $(\sin^2 x-\cos^2 x)[(\sin^2 x+\cos^2 x)^2$

$$=\int \frac{-2\sin^2 x\cos^2 x]}{1-2\sin^2 x\cos^2 x}$$

$$=\int \frac{(\sin^2 x-\cos^2 x)(1-2\sin^2 x\cos^2 x)}{1-2\sin^2 x\cos^2 x}dx$$

$$=-\int \cos 2x dx=-\frac{1}{2}\sin 2x+C$$

28. (d) Let $I=\int \cos^n x\sin x dx$

Put $\cos x=t$

$-\sin x dx=dt$

$$\therefore \quad I=-\int t^n dt=-\frac{t^{n+1}}{n+1}+C$$

$$=-\frac{\cos^{n+1} x}{n+1}+C=-\frac{\cos^6 x}{6}+C$$

$\therefore n+1=6$ or $n=5$

29. (b) Let $I=\int \sin^3 x\cos^3 x dx$. Then, $I=\frac{1}{8}\int (2\sin x\cos x)^3 dx \Rightarrow I=\frac{1}{8}\int \sin^3 2x dx \Rightarrow I=\frac{1}{8}\int \frac{3\sin 2x-\sin 6x}{4}dx$

$$\Rightarrow I=\frac{1}{32}\int (3\sin 2x-\sin 6x)dx=\frac{1}{32}\left\{-\frac{3}{2}\cos 2x+\frac{1}{6}\cos 6x\right\}+C$$

30. (b) Let $I=\int \frac{1}{\sqrt{\sin^3 x\cos^5 x}}dx$

$\Rightarrow I=\int \frac{1}{\sin^{3/2} x\cos^{5/2} x}dx \Rightarrow I=\int \frac{\sec^4 x}{\tan^{3/2} x}dx$ [Dividing numerator and denominator by $\cos^4 x$]

$$\Rightarrow \quad I=\int \frac{(1+\tan^2 x)}{\tan^{3/2} x}\sec^2 x dx$$

Putting $\tan x=t \Rightarrow \sec^2 x dx=dt$

$$I=\int \frac{1+t^2}{t^{3/2}}dt \Rightarrow I=\int (t^{-3/2}+t^{1/2})dt=\frac{-2}{\sqrt{t}}+\frac{t^{3/2}}{3/2}+C$$

$$=-\frac{2}{\sqrt{\tan x}}+\frac{2}{3}(\tan x)^{3/2}+C$$

31. (b)

32. (b) $\int \frac{\sin x}{\sin (x-\alpha)}dx=\int \frac{\sin (x-\alpha+\alpha)}{\sin (x-\alpha)}dx$

$$=\int \frac{\sin (x-\alpha)\cos \alpha+\cos (x-\alpha)\sin \alpha}{\sin (x-\alpha)}dx$$

$= \int \{\cos\ \alpha + \sin\ \alpha \cot\ (x-\alpha)\}dx$
$= (\cos\ \alpha)x + (\sin\ \alpha)\log\ \sin\ (x-\alpha) + C$

33. (b)

$\therefore A = \cos\ \alpha, B = \sin\ \alpha$

34. (b) $\int \frac{(x-x^3)^{1/3}}{x^4} dx = \int \frac{1}{x^3}\left(\frac{1}{x^2}-1\right)^{1/3} dx$

$= \frac{-1}{2}\int t^{1/3}dt \quad \left[\quad \text{Putting } \frac{1}{x^2} - 1 = t \Rightarrow \frac{-2}{x^3}dx = dt\right]$

$$= \frac{-1}{2} \cdot \frac{t^{4/3}}{4/3} + C = \frac{-3}{8}\left(\frac{1}{x^2}-1\right)^{4/3} + C$$

35. (c) $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} dx$

Let $e^{2x} + e^{-2x} = t$

$$\Rightarrow 2e^{2x} - 2e^{-2x} = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{2(e^{2x}-e^{-2x})}$$

$$\therefore \int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} dx = \int \frac{e^{2x}-e^{-2x}}{t} = \frac{dt}{2[e^{2x}-e^{-2x}]}$$

$$= \frac{1}{2}\int \frac{1}{t}dt = \frac{1}{2}\log\ |t| + C = \frac{1}{2}\log|e^{2x}+e^{-2x}| + C$$

36. (a) $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}} dx = g(x) + c$

Put $x = \cos\ 2t$

$dx = -2\sin\ 2t.\mathrm{dt}$

$$= \int \frac{1}{\cos\ 2t}\tan\ t(-4\sin\ t\cos\ t)dt$$

$$= \int \frac{1}{\cos\ 2t}(-4\sin^2\ t)dt = -2\int \frac{1-\cos\ 2t}{\cos\ 2t}dt$$

$$= -\frac{2}{2}\ln\ |\sec\ 2t + \tan\ 2t| + 2t + c$$

$$= \ln\ |\sec\ 2t - \tan\ 2t| + 2t + c$$

$$= \ln\ \left|\frac{1-\sin\ 2t}{\cos\ 2t}\right| + \cos^{-1}\ x + c$$

Let $g(x) = \ln\ \left|\frac{1-\sqrt{1-x^2}}{x}\right| + \cos^{-1}\ x + c$ (say)

$\therefore g(1) = 0 \Rightarrow \mathrm{c} = 0$

Now $g(x) = \ln\ \left|\frac{1-\sqrt{1-x^2}}{x}\right| + \cos^{-1}\ x$

$$g\left(\frac{1}{2}\right) = \ln\ |2-\sqrt{3}| + \frac{\pi}{3}$$

After checking options we get

$$g\left(\frac{1}{2}\right) = \ln\ \left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right| + \frac{\pi}{3}$$

37. (b) Let $\sec\ x = t \Rightarrow \tan\ n \cdot \sec\ xdx = dt$

$$\therefore \mathrm{I} = \int \frac{1}{t^2+(2)^2}dt = \log\ \left|t + \sqrt{t^2+(2)^2}\right| + c$$

$= \log \left|\sec x + \sqrt{\sec^2 x + 4}\right| + c$

38. (a) $I = \int \frac{x}{1-(x^2)^2} dx$ · Let $x^2 = t \Rightarrow xdx = \frac{1}{2} dt$

$I = \frac{1}{2} \int \frac{1}{1-t^2} \mathrm{dt} = \frac{1}{2} \times \frac{1}{2} \log \left|\frac{1+\mathrm{t}}{1-\mathrm{t}}\right| + \mathrm{c}$

$= \frac{1}{4} \log \left|\frac{1+\mathrm{x}^2}{1-\mathrm{x}^2}\right| + \mathrm{c}$

39. (b) Put $x - \alpha = t^2 \Rightarrow dx = 2tdt$

$$\therefore I = 2 \int \frac{tdt}{\sqrt{t^2(\beta - \alpha - t^2)}}$$

$$= 2 \int \frac{dt}{\sqrt{(\beta - \alpha) - t^2}} = 2\sin^{-1} \frac{t}{\sqrt{\beta - \alpha}} + C$$

$$= 2\sin^{-1} \sqrt{\frac{x - \alpha}{\beta - \alpha}} + C$$

40. (c) Let $I = \int \frac{x^3 + x}{x^4 - 9} dx$. Then,

$I = \int \frac{x^3}{x^4-9} dx + \int \frac{x}{x^4-9} dx = I_1 + I_2 + C$(say), where

$I_1 = \int \frac{x^3}{x^4-9} dx$ and $I_2 = \int \frac{x}{x^4-9} dx$

Putting $x^4 - 9 = t$ in $I_1 \Rightarrow 4x^3 dx = dt$, we get

$I_1 = \frac{1}{4} \int \frac{1}{t} dt = \frac{1}{4} \log |t| = \frac{1}{4} \log |x^4 - 9|$

$I_2 = \int \frac{x}{x^4 - 9} dx = \int \frac{x}{(x^2)^2 - 3^2} dx$

Putting $x^2 = t \Rightarrow 2xdx = dt$, we get

$I_2 = \frac{1}{2} \int \frac{dt}{t^2 - 3^2} = \frac{1}{2} \cdot \frac{1}{2 \times 3} \log \left|\frac{t-3}{t+3}\right| = \frac{1}{12} \log \left|\frac{x^2 - 3}{x^2 + 3}\right|$

Hence, $I = \frac{1}{4} \log |x^4 - 9| + \frac{1}{12} \log \left|\frac{x^2-3}{x^2+3}\right| + C$

41. (c) Let $I = \int \frac{1}{\sqrt{9+8x-x^2}} dx$. Then,

$I = \int \frac{1}{\sqrt{-\{x^2 - 8x - 9\}}} dx$

$I = \int \frac{1}{\sqrt{-\{x^2 - 8x + 16 - 25\}}} dx$

$\Rightarrow \mathrm{I} = \int \frac{1}{\sqrt{-\{(\mathrm{x}-4)^2 - 5^2\}}} \mathrm{dx} = \int \frac{1}{\sqrt{5^2 - (x-4)^2}} \mathrm{dx}$

$= \sin^{-1} \left(\frac{x-4}{5}\right) + C$

42. (c) Let $I = \int_0^{\pi} \frac{e^{\cos x} \sin x}{(1+\cos^2 x)(e^{\cos x} + e^{-\cos x})} dx$

Applying identity $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$$\mathrm{I} = \int_0^{\pi} \frac{e^{-\cos x}\sin x}{(1+\cos^2 x)(e^{-\cos x}+e^{\cos x})}dx$$

Add equations (i) and (ii), we get

$$2\mathrm{I} = \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

On putting $\cos x = t$, we get

$$2\mathrm{I} = 2\int_0^1 \frac{dt}{1+t^2} = (\tan^{-1} t)_0^1 = \frac{\pi}{4}$$

43. (a) Put $x = z^6 \Rightarrow dx = 6z^5 dz$

$$\therefore \quad \int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx = \int \frac{(z^6+z^4+z)6z^5dz}{.z^6(1+z^2)}$$

$$= 6\int \frac{z^5+z^3+1}{z^2+1}dz = 6\int \left(z^3+\frac{1}{z^2+1}\right)dz$$

$$= \frac{3}{2}z^4 + 6\tan^{-1} z + C = \frac{3}{2}x^{2/3} + 6\tan^{-1} x^{1/6} + C$$

44. (c) $I = \int \frac{1}{1+3\sin^2 x+8\cos^2 x}dx$

Dividing the numerator and denominator by $\cos^2 x$, we get

$$I = \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x + 8}dx$$

$$\Rightarrow I = \int \frac{\sec^2 x}{1+\tan^2 x + 3\tan^2 x + 8}dx = \int \frac{\sec^2 x}{4\tan^2 x + 9}dx$$

Putting $\tan x = t \Rightarrow \sec^2 x dx = dt$, we get

$$I = \int \frac{dt}{4t^2+9} = \frac{1}{4}\int \frac{dt}{t^2+(3/2)^2} = \frac{1}{4}\times\frac{1}{3/2}\tan^{-1}\left(\frac{t}{3/2}\right)+C$$

$$\Rightarrow \quad I = \frac{1}{6}\tan^{-1}\left(\frac{2t}{3}\right)+C = \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$$

45. (b) $I = \int \frac{1+1/x^2}{x^2+1+1/x^2}dx = \int \frac{d(x-1/x)}{(x-1/x)^2+3}$

$$= \frac{1}{\sqrt{3}}\tan^{-1} \frac{x-1/x}{\sqrt{3}} + C$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{(x^2-1)}{\sqrt{3}x}\right)+C$$

46. (a) Let $x = a\sin^2 \theta$

then $dx = 2a\sin \theta\cos \theta d\theta$

$$\therefore \quad \mathrm{I} = \int \frac{2a\sin \theta \cdot \cos \theta}{\sqrt{a\sin^2 \theta \cdot a\cos^2 \theta}}d\theta$$

$$= 2\int d\theta = 2\theta + c$$

$$= 2\sin^{-1}(\sqrt{x/a}) + c$$

47. (b) After dividing by $\cos^2 x$ to numerator and denominator of integration

$$I = \int \frac{\sec^2 x dx}{4\tan^2 x + 4\tan x + 5}$$

$$= \int \frac{\sec^2 x dx}{(2\tan x + 1)^2 + 4} = \frac{1}{22}\tan^{-1}\left(\frac{2\tan x + 1}{2}\right) + C$$

48. (a) Let $6x + 7 = \lambda \frac{d}{dx}(x-5)(x-4) + \mu$

i.e. $6x + 7 = \lambda(2x - 9) + \mu$ which

gives $\lambda = 3$ and $\mu = 34$

$$\therefore \int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx = \int \frac{3(2x-9)+34}{x^2-9x+20} dx$$

$$= 3\int (2x-9)(x^2-9x+20)^{-\frac{1}{2}} dx + 34\int \frac{dx}{\sqrt{x^2-9x+20}}$$

$$= 3\int \frac{(x^2-9x+20)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + 34\int \frac{dx}{\sqrt{x^2-9x+\frac{81}{4}-\frac{1}{4}}}$$

$$= 6\sqrt{x^2-9x+20} + 34\int \frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$= 6\sqrt{x^2-9x+20}$$

$$+34\log\left\{\left|x-\frac{9}{2}+\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right|\right\} + C$$

$$= 6\sqrt{x^2-9x+20} + 34\log\left|x+\sqrt{x^2-9x+20}-\frac{9}{2}\right| + C \therefore A = 6, B = 34.$$

49. (b) $I = \int \frac{dx}{x\sqrt{1-x^3}} = \int \frac{x^2 dx}{x^3\sqrt{1-x^3}}$

Put $1 - x^3 = t^2 \Rightarrow -3x^2 dx = 2t dt$

$$I = -\frac{2}{3}\int \frac{t dt}{(1-t^2)\cdot t} = \frac{2}{3}\int \frac{dt}{t^2-1} = \frac{2}{3}\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C$$

$$= \frac{1}{3}\ln\left[\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right] + C \therefore a = \frac{1}{3}, b = -1$$

50. (b) We have, $I = \int \frac{\sqrt{x}}{\sqrt{a^3-x^3}} dx$

$$I = \int \frac{\sqrt{x}}{\sqrt{(a^{3/2})^2-\left(-\frac{3}{2}\right)^2}} dx$$

Put, $x^{\frac{3}{2}} = t$

$$\Rightarrow \frac{3}{2}x^{\frac{1}{2}} dx = dt \Rightarrow \sqrt{x} dx = \frac{2}{3} dt$$

$$\therefore I=\frac{2}{3}\int \frac{dt}{\left(a^{\frac{3}{2}}\right)^2-t^2}=\frac{2}{3}\sin^{-1}\left(a^{\frac{t}{2}}\right)+C$$

$$=\frac{2}{3}\sin^{-1}\left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}}\right)+C=\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$$

51. (b) $I=\int\sqrt{\frac{a-x}{a+x}}dx=\int\sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}dx=\int\frac{a-x}{\sqrt{a^2-x^2}}dx$

$\Rightarrow I=\int\frac{a}{\sqrt{a^2-x^2}}dx-\int\frac{x}{\sqrt{a^2-x^2}}dx$

$\Rightarrow \mathrm{I}=\mathrm{a}\int\frac{1}{\sqrt{\mathrm{a}^2-\mathrm{x}^2}}\mathrm{dx}+\frac{1}{2}\int\frac{-2\mathrm{x}}{\sqrt{\mathrm{a}^2-\mathrm{x}^2}}\mathrm{dx}$

Putting $\mathrm{a}^2-\mathrm{x}^2=\mathrm{t}$, and $-2\mathrm{xdx}=\mathrm{dt}$, we get

$I=\mathrm{a}\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}\int\frac{dt}{\sqrt{t}}=\mathrm{a}\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right)+C$

$\Rightarrow I=\mathrm{a}\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{t}+C=a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}+C$

52. (a) $\because\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{\mathrm{x}}{\mathrm{a}}+\mathrm{c}$

But it is given that

$\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{3}+c$

$\therefore \mathrm{a}=3$

53. (b) $I=2\int\sqrt{x^2+\frac{9}{4}}dx=2\int\sqrt{x^2+\left(\frac{3}{2}\right)^2}dx=\frac{x}{2}\sqrt{4x^2+9}+\frac{9}{4}\log/2x+\sqrt{4x^2+9}/+C$

54. (a) $I=\int\sqrt{-(x^2-7x+10)dx}=\int\sqrt{\left(\frac{3}{2}\right)^2-\left(x-\frac{7}{2}\right)^2dx}=\frac{1}{4}(2x-7)\sqrt{7x-10-x^2}+\frac{9}{8}\sin^{-1}\left(\frac{2x-7}{3}\right)+c$

55. (c) $\int\sqrt{x^2-8x+7}dx=\int\sqrt{(x-4)^2-(3)^2}dx$

56. (b)

Now apply formula of $\int\sqrt{x^2-a^2}dx.$

57. (a) $\int\frac{x}{(x-1)(x^2+4)}dx$

Let $\frac{x}{(x-1)(x^2+4)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+4}$

or $x=A(x^2+4)+(Bx+C)(x-1)$

Solving, we obtain

$A=\frac{1}{5}, B=-\frac{1}{5}$ and $C=\frac{4}{5}$.

Substituting the values of A, B, and C in equation (i), we obtain

$$\frac{x}{(x-1)(x^2+4)}=\frac{1}{5(x-1)}+\frac{-\frac{1}{5}x+\frac{4}{5}}{x^2+4}$$

$$=\frac{1}{5(x-1)}-\frac{1}{5}\frac{(x-4)}{(x^2+4)}$$

or $\quad I=\frac{1}{5}\int \frac{1}{x-1}dx-\frac{1}{5}\int \frac{x-4}{x^2+4}dx$

$$=\frac{1}{5}\int \frac{1}{x-1}dx-\frac{1}{10}\int \frac{2x}{x^2+4}dx+\frac{4}{5}\int \frac{1}{x^2+4}dx$$

$$=\frac{1}{5}\log\,|x-1|-\frac{1}{10}\log\,(x^2+4)+\frac{4}{5}\times\frac{1}{2}\tan^{-1}\,\frac{x}{2}+C$$

$$=\frac{1}{5}\log\,|x-1|-\frac{1}{10}\log\,(x^2+4)+\frac{2}{5}\tan^{-1}\,\left(\frac{x}{2}\right)+C$$

58. (b) Let $\frac{1}{(x-1)(x+2)(2x+3)}=\frac{A_1}{x-1}+\frac{A_2}{x+2}+\frac{A_3}{2x+3}$

$$A_1=\frac{1}{(1+2)(2.1+3)}=\frac{1}{15};A_2=\frac{1}{(-2-1)(-2.2+3)}=\frac{1}{3}$$

$$A_3=\frac{1}{\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}+2\right)}=\frac{1}{-\frac{5}{4}}=-\frac{4}{5}$$

Now $I=\int \frac{dx}{(x-1)(x+2)(2x+3)}$

$$=\int \left[\frac{1}{15(x-1)}+\frac{1}{3(x+2)}-\frac{4}{5(2x+3)}\right]dx$$

$$=\frac{1}{15}\log\,|x-1|+\frac{1}{3}\log\,|x+2|-\frac{2}{5}\log\,|2x+3|+C$$

59. (d) We have,

$$\frac{3x+1}{(x-3)(x-5)}=\frac{-5}{x-3}+\frac{B}{x-5}$$

$$3x+1=-5(x-5)+B(x-3)$$

Put $x=5$

$$3(5)+1=B(5-3)$$

$16=2B$ or $B=8$

60. (c) Let $\frac{1}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$

Comparing both sides we get $A=\frac{1}{4},B=-\frac{1}{4}$ and $C=-\frac{1}{2}$

$$\therefore I=\frac{1}{4}\int \frac{1}{x-1}dx\frac{1}{4}\int \frac{1}{x+1}dx-\frac{1}{2}\int \frac{1}{(x+1)^2}dx$$

61. (a) Here since the highest powers of x in numerator and denominator are equal and coefficients of x^2 are also equal, therefore $\int \frac{x^2+1}{(x-1)(x-2)}\equiv 1+\frac{A}{x-1}+\frac{B}{x-2}$

On solving we get $A=-2,B=5$

Thus $\int \frac{x^2+1}{(x-1)(x-2)}\equiv 1-\frac{2}{x-1}+\frac{5}{x-2}$

The above method is used to obtain the value of constant corresponding to non-repeated linear factor in the denominator.

Now, $I=\int \left(1-\frac{2}{x-1}+\frac{5}{x-2}\right)dx$

$$=x-2\log\,(x-1)+5\log\,(x-2)+C$$

$$=x+\log\,\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$$

62. (d) We have, $I=\int \frac{1}{(\sin\,x+4)(\sin\,x-1)}dx$

$$= \frac{1}{5}\int \frac{(\sin\ x+4)-(\sin\ x-1)}{(\sin\ x+4)(\sin\ x-1)}dx$$
$$= \frac{1}{5}\int \frac{1}{\sin\ x-1}dx - \frac{1}{5}\int \frac{1}{\sin\ x+4}dx$$
$$= \frac{1}{5}\int \frac{\sec^2\ \frac{x}{2}}{2\tan\ \frac{x}{2}-1-\tan^2\ \frac{x}{2}}dx$$
$$-\frac{1}{5}\int \frac{\sec^2\ \frac{x}{2}}{2\tan\ \frac{x}{2}+4+4\tan^2\ \frac{x}{2}}dx$$

Put, $\tan\ \frac{x}{2} = t$

$$\Rightarrow \quad \sec^2\ \frac{x}{2}dx = 2dt$$
$$\therefore \mathrm{I} = \frac{1}{5}\int \frac{2\mathrm{dt}}{2\mathrm{t}-1-\mathrm{t}^2} - \frac{1}{5}\int \frac{2\mathrm{dt}}{[2\mathrm{t}+4(1+\mathrm{t}^2)]}$$
$$\therefore I = -\frac{2}{5}\int \frac{dt}{t^2-2t+1} - \frac{1}{10}\int \frac{dt}{t^2+\frac{1}{2}t+1}$$
$$= -\frac{2}{5}\int \frac{1}{(t-1)^2}dt - \frac{1}{10}\int \frac{dt}{\left(t+\frac{1}{4}\right)^2+\left(\frac{\sqrt{15}}{4}\right)^2}$$
$$= \frac{2}{5}\cdot\frac{1}{t-1} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4t+1}{\sqrt{15}}\right)+C$$
$$= \frac{2}{5}\frac{1}{\tan\ \frac{x}{2}-1} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4\tan\ \frac{x}{2}+1}{\sqrt{15}}\right)+C$$

$$\mathrm{I} = \mathrm{A}\frac{1}{\left(\tan\ \frac{\mathrm{x}}{2}-1\right)} + \mathrm{B}\tan^{-1}\ [\mathrm{f(x)}] + \mathrm{C}$$

From eqs. (i) and (ii), we get

$$A = \frac{2}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4\tan\ \frac{x}{2}+1}{\sqrt{15}}$$

63. (a) Let $I = \int \frac{1-\cos\ x}{\cos\ x(1+\cos\ x)}dx$

Let $\cos\ x = y \Rightarrow \frac{1-\cos\ x}{\cos\ x(1+\cos\ x)} = \frac{1-y}{y(1+y)}$

Now $\frac{1-y}{y(1+y)} = \frac{A}{y} + \frac{B}{1+y}$

$\Rightarrow 1-\mathrm{y} = \mathrm{A}(1+\mathrm{y}) + \mathrm{By}$

Put $y = 0$ in (i), we get $A = 1$.

Put $y = -1$ in (i), we get $B = -2$

Substituting the values of A and B in (i), we obtain

$$\frac{1-y}{y(1+y)}=\frac{1}{y}-\frac{2}{1+y}$$

$$\Rightarrow \quad \frac{1-\cos x}{\cos x(1+\cos x)}=\frac{1}{\cos x}-\frac{2}{1+\cos x}[\because y=\cos x]$$

$$\therefore \quad I=\int \frac{1-\cos x}{\cos x(1+\cos x)} d x=\int \frac{1}{\cos x} d x-\int \frac{2}{1+\cos x} d x$$

$$\Rightarrow \quad I=\int \sec x d x-\int \frac{1}{\cos ^2(x / 2)} d x$$

$$=\int \sec x d x-\int \sec ^2(x / 2) d x$$

$$\Rightarrow \quad I=\log |\sec x+\tan x|-2 \tan (x / 2)+C$$

64.

(d) $\frac{3x+4}{x^3-2x-4}=\frac{3x+4}{(x-2)(x^2+2x+2)}$

$$=\frac{A}{x-2}+\frac{Bx+C}{x^2+2x+2}$$

$\Rightarrow \quad 3x+4=A(x^2+2x+2)+(Bx+C)(x-2)$

$\therefore \quad A+B=0$

$$2A-2B+C=3$$
$$2A-2C=4$$

$\Rightarrow A=1, B=C=-1$

$\therefore \int \frac{3x+4}{x^3-2x-4} dx=\int \frac{dx}{x-2}-\frac{1}{2} \int \frac{2x+2}{x^2+2x+2} dx$

$=\log_e |x-2|-\frac{1}{2} \log |x^2+2x+2|+C$

$\Rightarrow k=-\frac{1}{2}$ and $f(x)=|x^2+2x+2|$

65. (b) We have, $\frac{x^3-1}{x^3+x}=1-\frac{1}{x}+\frac{x}{x^2+1}-\frac{1}{x^2+1}$

[Partial fraction]

$\therefore \quad \int \frac{x^3-1}{x^2+x} dx=x-\log x+\frac{1}{2} \log (1+x^2)-\tan^{-1} x+C$

66. (d)

67. (a) We have, $I=\int 1 \cdot \tan^{-1} \sqrt{x} dx$

Using by parts,

$I=\tan^{-1} \sqrt{x}.(x)-\int \frac{1}{1+x} \times \frac{1}{2 \sqrt{x}} \times x d x$

$=x \tan^{-1} \sqrt{x} \cdot-\int \frac{x}{(1+x) 2 \sqrt{x}} d x$

$=x \tan^{-1} \sqrt{x}-\int\left(\frac{1+x}{(1+x) 2 \sqrt{x}}-\frac{1}{(1+x) 2 \sqrt{x}}\right) d x$

$=x \tan^{-1} \sqrt{x}-\int \frac{dx}{2 \sqrt{x}}+\int \frac{dx}{2 \sqrt{x}(1+x)}$

$=x \tan^{-1} \sqrt{x}-\sqrt{x}+\tan^{-1} \sqrt{x}+C$

$=(x+1) \tan^{-1} \sqrt{x}-\sqrt{x}+C$

68. (b)

69. (c) $\int e^x \frac{(1+\sin x)}{(1+\cos x)} dx = \int e^x \left[\frac{1}{2}\sec^2 \frac{x}{2} + \tan \frac{x}{2}\right] dx$

$$= \frac{1}{2}\int e^x \sec^2 \frac{x}{2} dx + \int e^x \tan \frac{x}{2} dx$$

$$= e^x \tan \frac{x}{2} + C$$

But $I = e^x f(x) + C$ (given)

$\therefore \mathrm{f(x)} = \tan \frac{\mathrm{x}}{2}$

70.

(b) $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx = \int e^x \left(\frac{1-\sin x}{2\sin^2 \frac{x}{2}}\right) dx$

$= \int \mathrm{e^x} \left(\frac{1}{2}\mathrm{cosec}^2 \frac{\mathrm{x}}{2} - \cot \frac{\mathrm{x}}{2}\right) \mathrm{dx}$

$= \frac{1}{2}\int e^x \mathrm{cosec}^2 \frac{x}{2} dx - e^x \cot \frac{x}{2} - \frac{1}{2}\int e^x \mathrm{cosec}^2 \frac{x}{2} dx + C$

$= -e^x \cot \frac{x}{2} + C$

71. (b) Let $\sin^{-1} x = \theta \Rightarrow x = \sin \theta$

$\Rightarrow \mathrm{dx} = \cos \theta \mathrm{d}\theta$

$\therefore I = \int \theta \cos \theta d\theta = \theta \int \cos \theta d\theta + \int \left(\frac{d\theta}{d\theta}\int \cos \theta d\theta\right) d\theta$

$= \theta \sin \theta + \int \sin \theta d\theta = \theta \sin \theta - \cos \theta + c$

$= x \sin^{-1} x - \sqrt{1-x^2} + c.$

72. (b) Let $\sin^{-1} \mathrm{x} = \mathrm{t} \Rightarrow \frac{1}{\sqrt{1-\mathrm{x}^2}} \mathrm{dx} = \mathrm{dt}$ and $\mathrm{x} = \mathrm{sint}$

$\therefore \mathrm{I} = \int + \sin \mathrm{tdt} = \mathrm{t}\int \sin \mathrm{tdt} - \int \left(\frac{\mathrm{dt}}{\mathrm{dt}}\int \sin \mathrm{tdt}\right) \mathrm{dt}$

$= -\mathrm{tcos}\ \mathrm{t} + \int \cos \mathrm{t}dt = \theta - \mathrm{tcos}\ \mathrm{t} + \sin \mathrm{t} + \mathrm{c}$

$= \mathrm{x} - \sqrt{1-\mathrm{x}^2}\sin^{-1} \mathrm{x} + \mathrm{c}$

73. (a) $I = \int x e^x dx = x\int e^x dx - \int \left(\frac{dx}{dx}\int e^x dx\right) dx$

$= \mathrm{xe^x} - \int \mathrm{e^x dx} = \mathrm{xe^x} - \mathrm{e^x} + \mathrm{c}$

74. (b) $I = \int 1 \cdot \log x dx = \log x \int 1 dx - \int \left(\frac{d\log x}{dx}\int 1 dx\right) dx = x\log x - \int 1 dx = x\log x - x + c$

(b) $\int x \sin x \sec^3 x dx = \int x \sin x \frac{1}{\cos^3 x} dx$

$$= \int x \tan x \cdot \sec^2 x dx$$

Now put $\tan x = t \Rightarrow \sec^2 x dx = dt$ and $x = \tan^{-1} t$, then it reduces to

$$\int \tan^{-1} t \cdot t dt = \frac{t^2}{2}\tan^{-1} t - \frac{1}{2}t + \frac{1}{2}\tan^{-1} t$$

$$= \frac{x(\sec^2 x - 1)}{2} - \frac{1}{2}\tan x + \frac{1}{2}x = \frac{1}{2}[x\sec^2 x - \tan x] + c.$$

76. (c) Let $I = \int x \sec^2 x dx$

$\Rightarrow I = x\int \sec^2 x dx - \int \left[\frac{d}{dx}(x)\int \sec^2 x\right] dx$

$\Rightarrow \mathrm{I} = \mathrm{x}\tan\ \mathrm{x} - \int \tan\ \mathrm{x}dx$

$\Rightarrow \mathrm{I} = \mathrm{x}\tan\ \mathrm{x} - (-\log\ \cos\ \mathrm{x}) + \mathrm{c}$

where ' c ' is a constant of integration.

$\Rightarrow \mathrm{I} = \mathrm{x}\tan\ \mathrm{x} + \log\ \cos\ \mathrm{x} + \mathrm{c}$

77. (c) Let the given integration be

$$I = \int e^x(1+\tan^2\ x+\tan\ x)dx$$

$$\Rightarrow I = \int e^x(\sec^2\ x+\tan\ x)dx$$

$$= \int e^x\tan\ xdx + \int e^x\sec^2\ xdx$$

$$= \tan\ x\cdot\int e^x dx - \int\left(\frac{d}{dx}\tan\ x\int e^x\right)dx + \int e^x\sec^2\ xdx$$

$= \tan\ x\cdot e^x - \int \sec^2\ xe^x dx + \int e^x\sec^2\ xdx$

$= \tan\ x\mathrm{e}^{\mathrm{x}} + \mathrm{c}$

78. (b) Let $I_3 = \int x^3e^{ax}dx = \frac{1}{a}x^3e^{ax} - \frac{3}{a}I_2$

$$= \frac{1}{a}(x^3e^{ax}) - \frac{3}{a}\left[\frac{1}{a}x^2e^{ax} - \frac{2}{a}I_1\right]$$

$$= \frac{1}{a}x^3e^{ax} - \frac{3}{a^2}x^2e^{ax} + \frac{6}{a^2}\left[\frac{1}{a}xe^{ax} - \frac{1}{a}I_0\right]$$

$$= \frac{1}{a}x^3e^{ax} - \frac{3}{a^2}x^2e^{ax} + \frac{6}{a^3}xe^{ax} - \frac{6}{a^3}\int e^{ax}dx$$

$$\left[\because I_0 = \int x^0e^{ax}dx = \int e^{ax}dx\right]$$

$$= \frac{1}{a}x^3e^{ax} - \frac{3}{a^2}x^2e^{ax} + \frac{6}{a^3}xe^{ax} - \frac{6}{a^4}e^{ax} + c$$

$$= \frac{1}{a^4}e^{ax}[a^3x^3 - 3a^2x^2 + 6ax - 6] + c$$

79. (c) $I = \int 2\sin\ x\cdot\cos\ x\cdot\log\ \cos\ xdx$

put $\log\ \cos\ \mathrm{x} = \mathrm{t}$ $\quad\therefore -\frac{\sin\ \mathrm{x}}{\cos\ \mathrm{x}}\mathrm{dx} = \mathrm{dt}$

$I = \int 2\sin\ x\cdot\cos\ x\cdot t\frac{\cos\ x}{-\sin\ x}dt$

$= -2\int \cos^2\ x\cdot tdt = -2\int t^{2t}dt$

$= -\mathrm{e}^{2\mathrm{t}} + \frac{1}{2}\mathrm{e}^{2\mathrm{t}} + \mathrm{C}$

$e^{2t}\left(\frac{1}{2} - t\right) + k = \cos^2\ x\left\{\frac{1}{2} - \log\ \cos\ x\right\} + C$

80. (b) Here, $a = 1, b = 2, f(x) = x^2, b - a = 1 = nh$

$\therefore\quad \int_1^2 x^2dx = \lim_{h\to\infty}\sum_{r=0}^{n-1} f(a+rh)$

$= \lim_{h\to 0}[h\{1^2 + (1+h)^2 + (1+2h)^2 + \cdots + (1+(n-1)h)^2\}]$

$$= \lim_{h\to 0}\left[h\left[\begin{matrix}(1^2+1^2+n \text{ times }) + h^2(1^2+2^2+\cdots ..\\ +(n-1)^2 + 2h(1+2+\cdots+(n-1))\end{matrix}\right\}\right]$$

$$= \lim_{h\to 0} h\left\{n + h^2\frac{(n-1)n(2n-1)}{6} + 2h\frac{(n-1)n}{2}\right\}$$

$$= \lim_{h\to 0}\left\{nh + \frac{(\mathrm{nh}-\mathrm{h})\mathrm{nh}(2\mathrm{nh}-\mathrm{h})}{6} + \frac{2(\mathrm{hn})(\mathrm{nh}-\mathrm{h})}{2}\right\}$$

$= 1 + \frac{1}{3} + 1 = \frac{7}{3}$ (as $\mathrm{n}\to\infty, \mathrm{h}\to 0$)

81. (c) We kniow that,

$$\int_0^4 \frac{1}{\sqrt{x^2+2x+3}}dx = \int_0^4 \frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}}dx$$

$$= \left[\log\left|x+1+\sqrt{(x+1)^2+(\sqrt{2})^2}\right|\right]_0^4$$

$$= \left[\log\left|x+1+\sqrt{x^2+2x+3}\right|\right]_0^4$$

$$= \log(5+\sqrt{16+8+3}) - \log(1+\sqrt{3})$$

$$= \log(5+3\sqrt{3}) - \log(1+\sqrt{3})$$

$$= \log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$$

82. (a) We have $A_{n+1} - A_n = \int_0^{\frac{\pi}{2}} \frac{\sin(2n+1)x - \sin(2n-1)x}{\sin x}dx = \int_0^{\frac{\pi}{2}} \frac{2\cos 2nx \sin x}{\sin x}dx = 2\int_0^{\frac{\pi}{2}} \cos 2n\pi dx = 0$

Again $B_{n+1} - B_n = \int_0^{\frac{\pi}{2}} \frac{\sin^2(n+1)x - \sin^2 nx}{\sin^2 x}dx$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin(2n+1)x\sin x}{\sin^2 x}dx = A_{n+1}$$

83. (d) Given $f'(x) = f(x) \Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrating; $\log f(x) = x + c \Rightarrow f(x) = e^{x+c}$

$$f(0) = 1 \Rightarrow f(x) = e^x$$

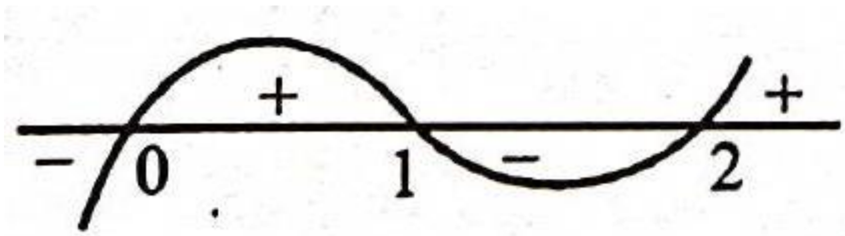
$$\therefore \int_0^1 f(x)g(x)dx = \int_0^1 e^x(x^2 - e^x)dx$$

$$= \int_0^1 x^2e^xdx - \int_0^1 e^{2x}dx$$

$$= [x^2e^x]_0^1 - 2[xe^x - e^x]_0^1 - \frac{1}{2}[e^{2x}]_0^1$$

$$= e - \left[\frac{e^2}{2} - \frac{1}{2}\right] - 2[e - e + 1] = e - \frac{e^2}{2} - \frac{3}{2}$$

84. (c) $\mathrm{f}'(\mathrm{x}) = \mathrm{x}(\mathrm{x}^2 - 3\mathrm{x} + 2) = \mathrm{x}(\mathrm{x}-1)(\mathrm{x}-2)$



$F(x)$ decrease in $[1,2]$ and increases in $[2,3]$ $\therefore$ Minimum of $\mathrm{f(x) = f(2)}$ and Maximum of $\mathrm{f(x) =}$ $\mathrm{Max\ \{f(1), f(3)\}}$

Now $f(x) = \left[\frac{t^4}{4} - t^3 + t^2\right]_1^x = \frac{x^4}{4} - x^3 + x^2 - \frac{1}{4}$

$\therefore \mathrm{f(1) = 0, f(2) =} -\frac{1}{4}$ and $\mathrm{f(3) = 2}$

$\therefore$ Range of $f(x) = \left[-\frac{1}{4}, 2\right]$

85. (a) $\int_0^{\pi/4} \sec^2\theta \sin\theta d\theta = \int_0^{\pi/4} \sec\theta \tan\theta d\theta$

$= [\sec\theta]_0^{\pi/4} = \sqrt{2} - 1.$

Now $\int_0^k \frac{dx}{(\sqrt{x}+\sqrt{x+k})} = \int_0^k \frac{\sqrt{x}-\sqrt{x+k}}{-k} dx$

$= -\frac{1}{k} \cdot \frac{2}{3}\left[x^{3/2} - (x+k)^{3/2}\right]_0^k$

$= -\frac{2}{3k}\left[k^{3/2} - (2k)^{3/2} + k^{3/2}\right]$

$= -\frac{2}{3}k^{1/2}[2 - 2\sqrt{2}] = \frac{4}{3}k^{1/2}(\sqrt{2} - 1)$

So, $\sqrt{2} - 1 = \frac{4}{3}k^{1/2}(\sqrt{2} - 1) \Rightarrow k = \frac{9}{16}$

86. (a) Let $\mathrm{I} = \int_{1/4}^{1/2} \frac{dx}{\sqrt{x - x^2}}$

$\therefore \mathrm{I} = \int_{1/4}^{1/2} \frac{dx}{\sqrt{\frac{1}{4} - \frac{1}{4} + x - x^2}}$

$\Rightarrow \mathrm{I} = \int_{1/4}^{1/2} \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}}$

$\Rightarrow \mathrm{I} = \left[\sin^{-1}\left(\frac{x - 1/2}{1/2}\right)\right]_{1/4}^{1/2} = [\sin^{-1}(2x - 1)]|_{1/4}^{1/2}$

$\Rightarrow \mathrm{I} = \sin^{-1}(1 - 1) - \sin^{-1}\left(2 \cdot \frac{1}{4} - 1\right)$

$\Rightarrow \mathrm{I} = 0 - \sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\sin\frac{\pi}{6}\right)$

$\Rightarrow \mathrm{I} = \frac{\pi}{6}$

87. (a) Let $I = \int_1^2 e^x \left[\frac{1}{x} - \frac{1}{x^2}\right] dx$

Consider $\int e^x \left[\frac{1}{x} - \frac{1}{x^2}\right] dx = \int e^x \cdot \frac{1}{x} dx - \int \frac{e^x}{x^2} dx$

We know $\int e^x(f(x) + f'(x))dx = e^x f(x)$

$$\therefore I=\int_1^2 e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx=\left[\frac{1}{x}e^x\right]_1^2$$
$$=\frac{1}{2}e^2-\frac{1}{1}e^1=e\left[\frac{e}{2}-1\right]$$

88. (a) Let $I=\int_0^1 x^2 e^x dx$

$=x^2\int_0^1 e^x dx-\int_0^1\left[\frac{d}{dx}(x^2)\int e^x\right]dx$

$\Rightarrow \mathrm{I}=[\mathrm{x}^2\cdot \mathrm{e}^{\mathrm{x}}]_0^1-\int_0^1 2\mathrm{x}\cdot \mathrm{e}^{\mathrm{x}}\mathrm{dx}$

$=1.e^1-2\int_0^1 xe^x dx=e-2\left[x\int_0^1 e^x dx-\int_0^1 e^x dx\right]$

$=\mathrm{e}-2[\mathrm{xe}^{\mathrm{x}}-\mathrm{e}^{\mathrm{x}}]_0^1=\mathrm{e}-2[\mathrm{e}-\mathrm{e}-(0-1)]=\mathrm{e}-2$

89. (a) $\int_1^3\left(\tan^{-1}\frac{\mathrm{x}}{\mathrm{x}^2+1}+\tan^{-1}\frac{\mathrm{x}^2+1}{\mathrm{x}}\right)\mathrm{dx}$

$=\int_1^3\left(\tan^{-1}\frac{x}{x^2+1}+\cot^{-1}\frac{x}{x^2+1}\right)dx=\int_1^3\frac{\pi}{2}dx=\pi$

[As $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$]

90. (c) $a_n-a_{n-1}=\int_0^{\frac{\pi}{2}}\frac{\sin^2 nx-\sin^2(n-1)x}{\sin x}dx$

$=\int_0^{\frac{\pi}{2}}\frac{\sin\{nx+(n-1)x\}\sin\{nx-(n-1)x\}}{\sin x}dx$

$=\int_0^{\frac{\pi}{2}}\frac{\sin(2nx-x)\sin x}{\sin x}dx=\int_0^{\frac{\pi}{2}}\sin(2n-1)x dx$

$=-\left[\frac{\cos(2n-1)x}{2n-1}\right]_0^{\frac{\pi}{2}}=-\left[\frac{\cos(2n-1)\frac{\pi}{2}}{2n-1}-\frac{\cos 0}{2n-1}\right]=\frac{1}{2n-1}$

$\therefore \mathrm{a}_2-\mathrm{a}_1=\frac{1}{3},\mathrm{a}_3-\mathrm{a}_2=\frac{1}{5},\mathrm{a}_4-\mathrm{a}_3=\frac{1}{7}$ ... ... etc.

$\therefore a_2-a_1,a_3-a_2,a_4-a_3$ ... . form an H.P.

91. (c) $\int_0^\infty\frac{\mathrm{dx}}{(\mathrm{x}^2+\mathrm{a}^2)(\mathrm{x}^2+\mathrm{b}^2)}=\frac{1}{\mathrm{b}^2-\mathrm{a}^2}\int_0^\infty\frac{(\mathrm{x}^2+\mathrm{b}^2)-(\mathrm{x}^2+\mathrm{a}^2)}{(\mathrm{x}^2+\mathrm{a}^2)(\mathrm{x}^2+\mathrm{b}^2)}$

$=\frac{1}{b^2-a^2}\int_0^\infty\left[\frac{1}{x^2+a^2}-\frac{1}{x^2+b^2}\right]dx$

$=\frac{1}{b^2-a^2}\left[\frac{1}{a}\tan^{-1}\frac{x}{a}-\frac{1}{b}\tan^{-1}\frac{x}{b}\right]_0^\infty$

$=\frac{1}{b^2-a^2}\left[\frac{\pi}{2a}-\frac{\pi}{2b}\right]=\frac{\pi}{2ab(a+b)}$

92. (b) Let $I=\int_0^{\pi/2}\sqrt{\tan x}+\sqrt{\cot x}dx$

$=\int_0^{\pi/2}\frac{\sin x+\cos x}{\sqrt{\sin x\cos x}}dx$

$=\int_0^{\pi/2}\frac{\sqrt{2}(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$

$=\int_0^{\pi/2}\frac{\sqrt{2}(\sin x+\cos x)}{\sqrt{1-(\sin x-\cos x)^2}}dx$

Put, $\sin x-\cos x=t$

$\Rightarrow (\cos\mathrm{x}+\sin\mathrm{x})\mathrm{dx}=\mathrm{dt}$

$= \int_{-1}^{1} \frac{\sqrt{2}}{\sqrt{1-t^2}} dt = \sqrt{2}\sin^{-1} t\Big|_{-1}^{1} = \sqrt{2}\left[\frac{\pi}{2}+\frac{\pi}{2}\right] = \pi\sqrt{2}.$

93. (c) $\int_{\log 2}^{x} \frac{du}{(e^u-1)^{1/2}} = \frac{\pi}{6}$

Put $e^u - 1 = t^2 \Rightarrow e^u du = 2tdt$

$$\Rightarrow \int_1^{\sqrt{e^x-1}} \frac{2t}{1+t^2} dt = \frac{\pi}{6}$$

$$\Rightarrow 2(\tan^{-1} t)_1^{\sqrt{e^x-1}} = \frac{\pi}{6} \Rightarrow \tan^{-1} \sqrt{e^x-1} - \frac{\pi}{4} = \frac{\pi}{12}$$

$\Rightarrow \sqrt{e^x-1} = \tan\frac{\pi}{3} \Rightarrow \sqrt{e^x-1} = \sqrt{3} \Rightarrow e^x = 4.$

94. (d) Let $I = \int_0^{\pi/2} \sin x \sin 2x dx = 2\int_0^{\pi/2} \sin^2 x \cos x dx$

Put $t = \sin x \Rightarrow dt = \cos x dx$

Now, $I = 2\int_0^1 t^2 dt = \frac{2}{3}[t^3]_0^1 = \frac{2}{3}$

95. (b) Let $I = \int_0^{\pi/2} \sqrt{\cos\theta}\sin^3\theta d\theta$ Put $t = \cos\theta \Rightarrow dt = -\sin\theta d\theta$, then

$$I = -\int_1^0 t^{1/2}(1-t^2)dt = \int_0^1 \left(t^{1/2} - t^{5/2}\right)dt$$

$$I = \left[\frac{2}{3}t^{3/2} - \frac{2}{7}t^{7/2}\right]_0^1 = \frac{8}{21}$$

96. (d) $f(e^3) = \int_1^{e^3} \frac{\ln t}{\ln 10(1+t)} dt$

$$f(\alpha) = \int_1^{\alpha} \frac{\ln t}{(\ln 10)(1+t)} dt$$

Let $t = \frac{1}{x} \Rightarrow x = \frac{1}{t}$

$$dt = \frac{-1}{x^2} dx$$

$$= \int_1^{\frac{1}{\alpha}} \frac{-\ln x}{(\ln 10)\left(1+\frac{1}{x}\right)}\left(-\frac{1}{x^2}\right) dx$$

$$\Rightarrow f(\alpha) = \frac{1}{\ln 10}\int_1^{\frac{1}{\alpha}} \frac{\ln x}{x(x+1)} dx$$

$$\Rightarrow f(e^{-3}) = \frac{1}{\ln 10}\int_1^{e^3} \frac{\ln t}{t(t+1)} dt$$

Add equation (i) and (ii)

$f(e^3) + f(e^{-3})$

$$= \left(\frac{1}{\ln 10}\right)_1^{e^3} \frac{\ln t}{(1+t)}\left[1+\frac{1}{t}\right] dt$$

$$= \left(\frac{1}{\ln 10}\right)_1^{3} \frac{\ln t}{t} dt$$

Let $\ln t = r; \frac{dt}{t} = dr$

$$= \frac{1}{\ln 10}\int_0^3 r dr = \left(\frac{1}{\ln 10}\right)\left(\frac{r^2}{2}\right)\Bigg|_0^3 = \left(\frac{1}{\log 10}\right)\frac{9}{2} = \frac{9}{2\log_e 10}$$

97. (b) Now we are given that the integral

$$I=\int_0^{\frac{\pi}{2}}\frac{dx}{3+2\sin\,x+\cos\,x}=\int_0^{\frac{\pi}{2}}\frac{\sec^2\,\frac{x}{2}\cdot dx}{2\tan^2\,\frac{x}{2}+4\tan\,\frac{x}{2}+4}$$

Put $\tan\,\frac{x}{2}=t\Rightarrow\frac{1}{2}\sec^2\,\left(\frac{x}{2}\right)dx=dt$

Now $I=\int_0^1\frac{dt}{(t+1)^2+1}=\tan^{-1}\,(x+1)|_0^1=\tan^{-1}\,2-\frac{\pi}{4}$

98. (a) Given integral is

$I=\int_0^1[2x-|3x^2-3x-2x+2|+1]dx$

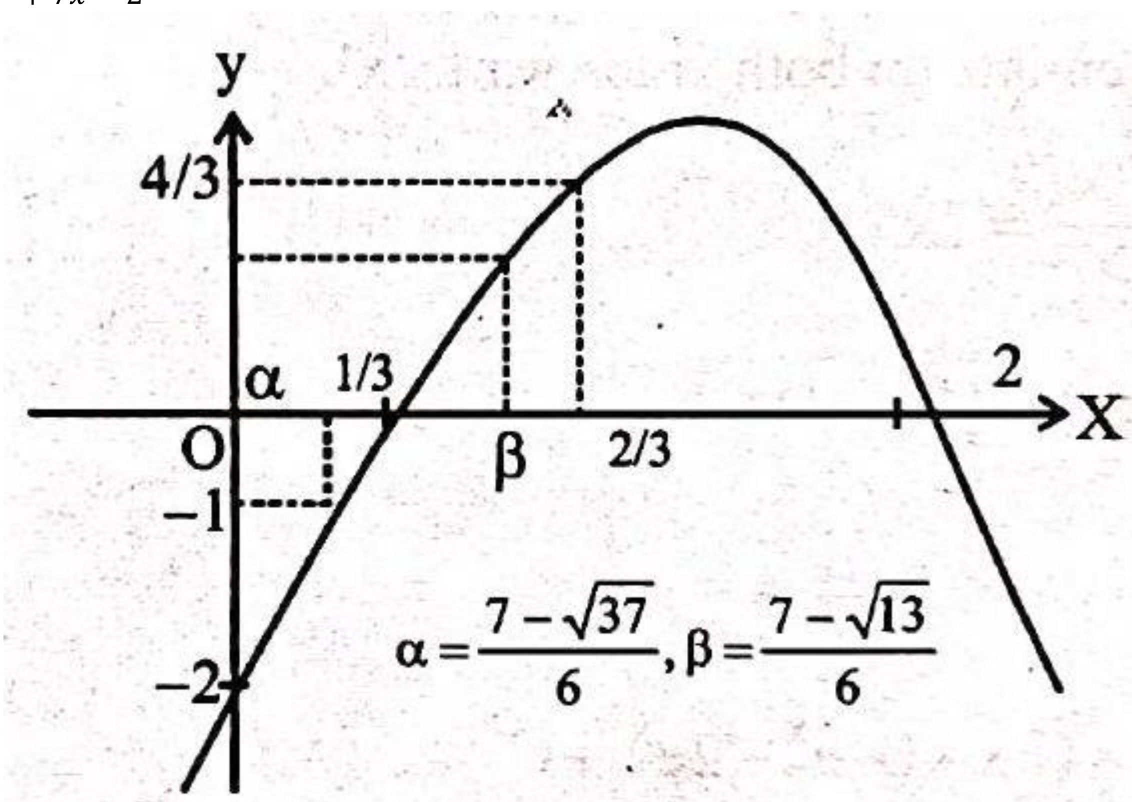
$I=\int_0^1[2x-|(3x-2)(x-1)|]dx+\int_0^1 1dx$

when $x<\frac{2}{3}$, then $(3x^2-5x-12)<0$ in second integral.

$I=\int_0^{\frac{2}{3}}\left[\left(2x-(3x^2-5x+2)\right)\right]dx+\int_{\frac{2}{3}}^1\left(2x+(3x^2-5x+2)\right)dx+1$

$I=\int_0^{\frac{2}{3}}[-3x^2+7x-2]dx+\int_{\frac{2}{3}}^1(3x^2-3x+2)dx+1$

$y=-3x^2+7x-2$

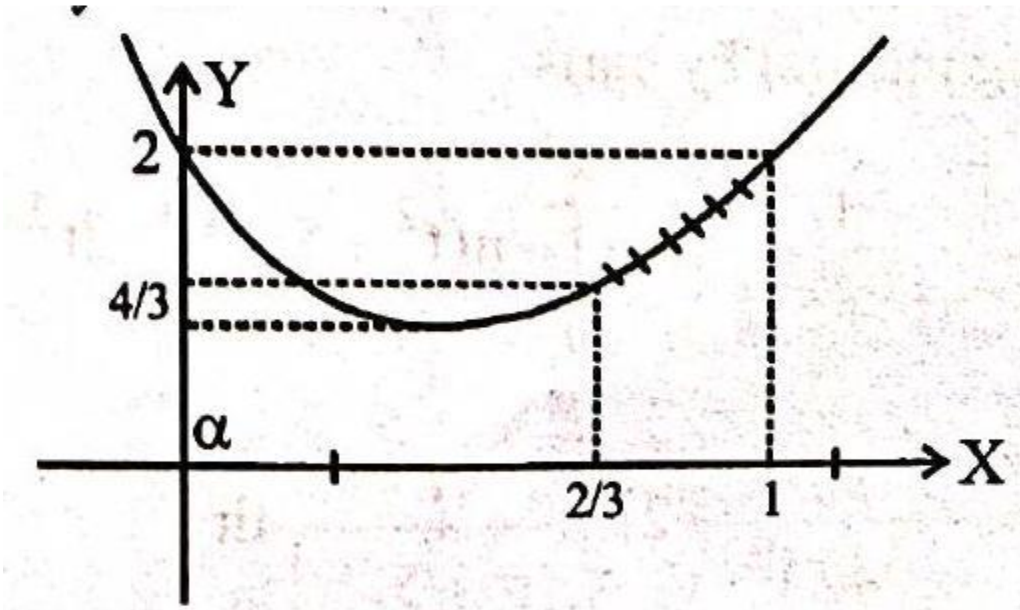


$\int_0^{\alpha}(-2)dx+\int_{\alpha}^{\frac{1}{3}}(-1)\mathrm{dx}+\int_{\frac{1}{3}}^{\beta}0\mathrm{dx}+\int_{\beta}^{\frac{2}{3}}1.\mathrm{dx}$

$=-2\alpha-\left(\frac{1}{3}-\alpha\right)+\frac{2}{3}-\beta=-\alpha-\beta+\frac{1}{3}$

$\mathrm{y}=3\mathrm{x}^2-3\mathrm{x}+2$

$y=3x-3x+2$



When $x \in \left(\frac{2}{3}, 1\right) 3x^2 - 3x + 2 \in \left(\frac{4}{3}, 2\right) [3x^2 - 3x + 2] = 1$

Therefore, $\int_{\frac{2}{3}}^{1} [3x^2 - 3x + 2] dx = 1\left(1 - \frac{2}{3}\right) = \frac{1}{3}$

Hence $I = \left(\frac{1}{3} - (\alpha + \beta)\right) + \left(\frac{1}{3}\right) + 1$

$$= \frac{5}{3} - \left(\frac{7 - \sqrt{37}}{6} + \frac{7 - \sqrt{13}}{6}\right)$$

$$= \frac{-2}{3} + \frac{\sqrt{37} + \sqrt{13}}{6} = \frac{\sqrt{37} + \sqrt{13} - 4}{6}$$

99. (a) $f(x) = e^x \cdot \int_0^x \frac{f'(t)}{e^t} dt$

$$f'(x) = e^x \cdot \int_0^x \frac{f'(t)}{e^t} dt + e^x \cdot \frac{f'(x)}{e^x}$$

$$-[(2x - 1) \cdot e^x + (x^2 - x + 1) \cdot e^x]$$

Now $\int_0^x \frac{f'(t)}{e^t} dt = x^2 + x$

Differentiate on both sides w.r.t. ' x '.

$$\Rightarrow \frac{f'(x)}{e^x} = 2x + 1$$

$$\Rightarrow f'(x) = (2x + 1) \cdot e^x$$

Now $f'(x) = 0 \Rightarrow x = -\frac{1}{2}$

$$f(x) = (2x + 1) \cdot e^x - 2e^x + C$$

$$\therefore f(0) = -1$$

$$-1 = 1 - 2 + C$$

$$\Rightarrow C = 0$$

Now $f(x) = e^x(2x - 1)$

$$\Rightarrow f\left(-\frac{1}{2}\right) = \frac{-2}{\sqrt{e}}$$

100. (a) Given that $I_n(x) = \int_0^x \frac{dt}{(t^2+5)^n}$

Applying integral by parts

$$I_n(x) = \left[\frac{t}{(t^2+5)^n}\right]_0^x - \int_0^x -n(t^2+5)^{-n-1} \cdot 2t^2$$

$$I_n(x) = \frac{x}{(x^2+5)^n} + 2n\int_0^x \frac{t^2}{(t^2+5)^{n+1}}dt$$

$$I_n(x) = \frac{x}{(x^2+5)^n} + 2n\int_0^x \frac{(t^2+5)-5}{t^2+5)^{n+1}}dt$$

$$I_n(x) = \frac{x}{(x^2+5)^n} + 2nI_n(x) - 10nI_{n+1}(x)$$

10n $I_{n+1}(x) + (1-2n)I_n(x) = \frac{x}{(x^2+5)^n}$

$$I_5\frac{d(I_5)}{dx} = \frac{1}{(x^2+5)^n}$$

Put $n = 5$ in (i) we get

101. (c) Consider $50I_6 - 9I_5 = I_5'$

Let $f(x) = 8\sin x - \sin 2x \Rightarrow f'(x) = 8\cos x - 2\cos 2x$

$$\Rightarrow f'(x) = -8\sin x + 4\sin 2x = -8\sin x(1-\cos x)$$

$\therefore$ $f''(x) < 0x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$\therefore$ $f'(x)$ is ↓ function

$$f'\left(\frac{\pi}{3}\right) < f'(x) < d'\left(\frac{\pi}{4}\right)$$

$$5 < f'(x) < \frac{8}{\sqrt{2}}$$

$$5 < f'(x) < 4\sqrt{2}$$

$$5x < f(x) < 4\sqrt{2}x$$

or $5 < \frac{f(x)}{x} < 4\sqrt{2}$

$$\int_{\pi/4}^{\pi/3} 5 < \int \frac{f(x)}{x} < \int_{\pi/4}^{\pi/3} 4\sqrt{2}$$

$$\int_{\pi/4}^{\pi/3} 5 < \int \frac{8\sin x - \sin 2x}{x} < \int_{\pi/4}^{\pi/3} 4\sqrt{2}$$

$$5[x]_{\pi/4}^{\pi/3} < \int \frac{(x)}{x} < [4\sqrt{2}]_{\pi/4}^{\pi/3}$$

$$\frac{5\pi}{12} < I < \frac{\sqrt{2}\pi}{3}$$

102. (b) $\lim_{x\to -1^+} a\sin\left(\pi\frac{[x]}{2}\right) + [2-x] = -a+2$

$$\lim_{x\to -1^-} a\sin\left(\pi\frac{[x]}{2}\right) + [2-x] = 0+3 = 0$$

$\lim_{x\to -1} g(x)$ exist when $a = -1$ Now,

$\int_0^4 g(x)dx = \int_0^1 g(x)dx + \int_1^2 g(x)dx + \int_2^3 g(x)dx$

$+\int_3^4 g(x)dx$

$= \int_0^1 (0+1)dx + \int_1^2 (-1+0)dx + \int_2^3 (0-1)dx + \int_3^4 (1-2)dx$

$= 1 - 1 - 1 - 1 = -2$

103. (a) We have,

$I = \int_0^{\pi/2} \frac{\cos\ x}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^3} dx$

$$= \int_0^{\pi/2} \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^3} dx = \int_0^{\pi/2} \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2} dx$$

Let $\cos\frac{x}{2} + \sin\frac{x}{2} = t$. Then,

$$\frac{1}{2}\left(-\sin\frac{x}{2} + \cos\frac{x}{2}\right) dx = dt$$

$$\Rightarrow \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) dx = 2dt$$

Also, $x = 0 \Rightarrow t = 1$ and $x = \frac{\pi}{2} \Rightarrow t = \sqrt{2}$

$$\therefore I = \int_1^{\sqrt{2}} \frac{2dt}{t^2} = 2\int_1^{\sqrt{2}} \frac{1}{t^2} \mathrm{dt}$$

$$= 2\left[-\frac{1}{\mathrm{t}}\right]_1^{\sqrt{2}} = 2\left[-\frac{1}{\sqrt{2}} + 1\right] = (2 - \sqrt{2})$$

104. (a) $I = \int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x} \sec^2\left(\frac{1}{3}e^{2x}\right) dx$

Put $e^{2x} = t \Rightarrow 2e^{2x} dx = dt$

When $x = \log\sqrt{\pi/2}, t = e^{2\log\sqrt{\pi/2}} = e^{\log\pi/2} = \frac{\pi}{2}$

When $x = \log\sqrt{\pi}, t = e^{2\log\sqrt{\pi}} = \pi$

$$\therefore I = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\sec^2\left(\frac{1}{3}t\right) dt = \frac{1}{2}\cdot\frac{1}{\frac{1}{3}}\left[\tan\frac{t}{3}\right]_{\pi/2}^{\pi}$$

$$= \frac{3}{2}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right] = \frac{3}{2}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = \sqrt{3}$$

105. (a) We have, $I = \int_0^{\pi} \frac{1}{5+4\cos\ x} dx$

$= \int_0^{\pi} \frac{1}{5 + 4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)} dx$

$$= \int_0^{\pi} \frac{1 + \tan^2\frac{x}{2}}{\left(1 + \tan^2\frac{x}{2}\right) + 4\left(1 - \tan^2\frac{x}{2}\right)} dx$$

$= \int_0^{\pi} \frac{1 + \tan^2\frac{x}{2}}{9 + \tan^2\frac{x}{2}} dx = \int_0^{\pi} \frac{\sec^2\frac{x}{2}}{9 + \tan^2\frac{x}{2}} dx$

Let $\tan\frac{\mathrm{x}}{2} = \mathrm{t} \Rightarrow \frac{1}{2}\sec^2\frac{\mathrm{x}}{2}\mathrm{dx} = \mathrm{dt}$

Also, $\mathrm{x} = 0 \Rightarrow \mathrm{t} = 0$ and $\mathrm{x} = \pi \Rightarrow \mathrm{t} = \infty$

$\therefore \quad \mathrm{I} = \int_0^{\infty} \frac{\mathrm{dt}}{9 + \mathrm{t}^2}$

$$\therefore \quad \mathrm{I} = 2\int_0^\infty \frac{\mathrm{dt}}{3^2 + \mathrm{t}^2}$$

$$\therefore \quad \mathrm{I} = \frac{2}{3}\left[\tan^{-1} \frac{\mathrm{t}}{3}\right]_0^\infty = \frac{2}{3}[\tan^{-1} \infty - \tan^{-1} 0]$$

$$= \frac{2}{3}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{3}$$

106. (c) Given $F(x) = f(x) + f\left(\frac{1}{x}\right)$,where

$$f(x) = \int_1^x \frac{\log t}{1+t} dt$$

$$\therefore F(e) = f(e) + f\left(\frac{1}{e}\right)$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt$$

Now for solving, $I = \int_1^{1/e} \frac{\log t}{1+t} dt$

$\therefore$ Put $\frac{1}{t} = z \Rightarrow -\frac{1}{t^2} dt = dz \Rightarrow dt = -\frac{dz}{z^2}$

and limit for $t = 1 \Rightarrow z = 1$ and for $t = 1/e \Rightarrow z = e$

$$\therefore I = \int_1^e \frac{\log\left(\frac{1}{z}\right)}{1+\frac{1}{z}}\left(-\frac{dz}{z^2}\right) = \int_1^e \frac{(\log 1 - \log z)\cdot z}{z+1}\left(-\frac{dz}{z^2}\right)$$

$$= \int_1^e -\frac{\log z}{(z+1)}\left(-\frac{dz}{z}\right) = \int_1^e \frac{\log z}{z(z+1)} dz$$

$$\therefore I = \int_1^e \frac{\log t}{t(t+1)} dt$$

Equation (i) becomes: $F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$

$$= \int_1^e \frac{t\cdot\log t + \log t}{t(1+t)} dt = \int_1^e \frac{(\log t)(t+1)}{t(1+t)} dt$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{t} dt$$

Let $\log t = x$

$$\therefore \frac{1}{t} dt = dx$$

[for limit $t = 1, x = 0$ and $t = e, x = \log e = 1$]

$$\therefore F(e) = \int_0^1 x dx \quad F(e) = \left[\frac{x^2}{2}\right]_0^1 \Rightarrow F(e) = \frac{1}{2}$$

107. (a)

108. (a) Put $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$

Also as $x = 0, \theta = 0$ and $x = 1, \theta = \frac{\pi}{4}$

Therefore, $\int_0^1 \tan^{-1} x dx = \int_0^{\pi/4} \theta \sec^2 \theta d\theta$

$$= \frac{\pi}{4} - \log\sqrt{2} = \frac{\pi}{4} - \frac{1}{2}\log 2.$$

109. (d) Given function is

$$f(x)=\frac{|x^3+x|}{(e^{x|x|}+1)}dx$$

Apply property

$\int_a^b f(x)dx=\int_a^b f(a+b-x)dx. \& \int_{-a}^{a} f(x)dx=2\int_0^a f(x)dx$

$$I=\int_{-2}^{2}\frac{|x^3+x|}{(e^{x|x|}+1)}dx$$

$$I=\int_{-2}^{2}\frac{|x^3+x|}{(e^{-x|x|}+1)}dx$$

Add (i) & (ii)

$$2I=2\int_0^2\left(\frac{|x^3+x|}{(e^{x|x|}+1)}+\frac{|-x^3-x|}{(e^{-x|-x|}+1)}\right)dx$$

$$I=\int_0^2\left(\frac{|x^3+x|}{(e^{x|x|}+1)}+\frac{|-x^3-x|}{(e^{-x|-x|}+1)}\right)dx$$

$$I=\int_0^2\left(\frac{|x^3+x|}{(e^{x|x|}+1)}+\frac{|x^3+x|}{(e^{-x|x|}+1)}\right)dx$$

$$I=\int_0^2\left(\frac{x^3+x}{(e^{x^2}+1)}+\frac{x^3+x}{(e^{-x^2}+1)}\right)dx$$

$$I=\int_0^2\left(\frac{x^3+x}{1+e^{x^2}+\frac{e^{x^2}(x^3+x)}{1+e^{x^2}}}\right)dx=\int_0^2 (x^3+x)dx$$

$$=\left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^2=4+2=6.$$

110. (b) Given slope of the tangent to a curve $y=f(x)$ at (x,y) be given by

$$\frac{dy}{dx}=2\tan\ x\cos\ x-2\tan\ x\cdot y$$

$$\Rightarrow\frac{dy}{dx}+(2\tan\ x)y=2\sin\ x$$

Now integrating factor $=e^{\int 2\tan\ xdx}=\frac{1}{\cos^2\ x}$

$\therefore$ solution is given by

$$y\left(\frac{1}{\cos^2\ x}\right)=\int\frac{2\sin\ x}{\cos^2\ x}dx$$

$$\Rightarrow y\sec^2\ x=\frac{2}{\cos\ x}+C$$

$$\Rightarrow y=2\cos\ x+C\cos^2\ x$$

$\because$ curve passes through $\left(\frac{\pi}{4},0\right)$

$\therefore$ from eqn. (i)

$$0=\sqrt{2}+\frac{C}{2}\Rightarrow C=-2\sqrt{2}$$

Now $f(x)=2\cos\ x-2\sqrt{2}\cos^2\ x$ is required curve

$\therefore \quad \int_0^{\pi/2} y dx = 2\int_0^{\pi/2} \cos\, x dx - 2\sqrt{2}\int_0^{\pi/2} \cos^2\, x dx$

$= [2\sin\, x]_0^{\pi/2} - 2\sqrt{2}\left[\frac{x}{2} + \frac{\sin\, 2x}{4}\right]_0^{\pi/2}$

$= 2 - \frac{\pi}{\sqrt{2}}$ is required answer

111. (c) We are given that

$f(x) + f(x+k) = n \forall x \in R$

or $\Rightarrow f(x+k) + f(x+2k) = n$

$\Rightarrow f(x+2k) + f(x+3k) = n$

Now from (i)

$f(x+k) = n - f(x)$

Now from eq (ii)

$n - f(x) + f(x+2k) = n$

$\Rightarrow f(x) = f(x+2k) \quad \forall \mathrm{x} \in \mathrm{R}$

$\Rightarrow f(x)$ is periodic with period $2k$

$I_1 = \int_0^{4nk} f(x) dx = 2n\int_0^{2k} f(x) dx$

$I_2 = \int_{-k}^{3k} f(x) dx = 2\int_0^{2k} f(x) dx$

$\because$ we are given that

$f(x) + f(x+k) = n$

$\Rightarrow \int_0^k f(x) dx + \int_0^k f(x+k) dx = nk$

$\Rightarrow \int_0^k f(x) dx + \int_k^{2k} f(x) dx = nk$

$\Rightarrow \int_0^{2k} f(x) dx = nk$

$\Rightarrow I_1 = 2n^2 k, I_2 = 2nk$

$\Rightarrow I_1 + nI_2 = 4n^2 k$

112. (b) We are given that

$g(x) = \int_x^{\pi/4} (f'(t)\sec\, t + \tan\, t\sec\, t f(t)) dt$

$\Rightarrow g(x) = \int_x^{\pi/4} d(f(t)\cdot\sec\, t) = f(t)\sec\, t|_x^{\pi/4}$

$\Rightarrow g(x) = f\left(\frac{\pi}{4}\right)\sec\, \frac{\pi}{4} - f(x)\cdot\sec\, x$

$\Rightarrow g(x) = 2 - f(x)\sec\, x = 2 - \left(\frac{f(x)}{\cos\, x}\right)$

Now $\lim_{x\to\left(\frac{\pi}{2}\right)^-} g(x) = 2 - \lim_{x\to\left(\frac{\pi}{2}\right)^-} \left(\frac{f(x)}{\cos\, x}\right) \left[\frac{0}{0}\right.$ form]

By L'Hopital Rule

$= 2 - \lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{f'(x)}{(-\sin\, x)}$

$= 2 + \frac{f'\left(\frac{\pi}{2}\right)}{\sin\, \frac{\pi}{2}} = 2 + \frac{1}{1} = 3$

113. (c) Given integral is $\int_0^1 [-8x^2 + 6x - 1] dx$, Take $f(x) = 0$,

Then, $x = \frac{1}{4}, \frac{1}{2}$

Put $x = 1$ in $f(x)$ then $y = -3$.

Now, find the values of x for the values of $y = -1 \& -2$

$$= \int_0^{1/4} -1dx + \int_{1/4}^{1/2} 0dx + \int_{1/2}^{3/4} -1dx + \int_{3/4}^{\frac{3+\sqrt{17}}{8}} -2dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} -3dx$$

$$= -[x]_0^{1/4} + 0 - [x]_{1/2}^{3/4} + -2[x]_{3/4}^{\frac{3+\sqrt{17}}{8}} - 3[x]_{\frac{3+\sqrt{17}}{8}}^{1}$$

Apply the limit,

$$= -\left(\frac{1}{4} - 0\right) - \left(\frac{3}{4} - \frac{1}{2}\right) - 2\left(\frac{3+\sqrt{17}}{8} - \frac{3}{4}\right) - 3\left(1 - \frac{3+\sqrt{17}}{8}\right)$$

$$= -\frac{1}{4} - \frac{1}{4} + \frac{-6-2\sqrt{17}}{8} + \frac{3}{2} - 3 + \frac{9+3\sqrt{17}}{8}$$

$$= \frac{\sqrt{17}-13}{8}$$

114. (c) $\int_0^2 \sqrt{2x}dx - \int_0^2 \sqrt{1-(x-1)^2}dx$

$$= \int_0^2 \left(1 - \frac{y^2}{2}\right) dy - \int_0^1 \sqrt{1-y^2}dy + 1 + \mathrm{I}$$

$$= \frac{8}{3} - 2\int_0^1 \sqrt{1-y^2}dy = \frac{2}{3} + 1 - \int_0^1 \sqrt{1-y^2}dy + I$$

$$\Rightarrow I = 1 - \int_0^1 \sqrt{1-y^2}dy$$

$$\Rightarrow \quad I = \int_0^1 \left(1 - \sqrt{1-y^2}\right) dy$$

EXERCISE - 2

1. (a) Suppose

$$I = \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$$

$$= \int \frac{(2\cos^2 x - 1) - (2\cos^2 \theta - 1)}{\cos x - \cos \theta} dx$$

$$= 2\int \frac{(\cos x + \cos \theta)(\cos x - \cos \theta)}{(\cos x - \cos \theta)} dx$$

$$= 2\int (\cos x + \cos \theta) dx$$

$$= 2(\sin x + x\cos \theta) + C$$

2. (c) Suppose,

$$I = \int \frac{dx}{\sin(x-a)\sin(x-b)}$$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin(b-a)}{\sin(x-a)\sin(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin[(x-a)-(x-b)]}{\sin(x-a)\sin(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a)\cos(x-b) - \cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int (\cot(x-b) - \cot(x-a)) dx$$

$$= \frac{1}{\sin(b-a)} (\log|\sin(x-b)| - \log|\sin n(x-a)|) + C$$

$$= \operatorname{cosec}(b-a) \log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + C$$

3. (d) We have, $I = \int \frac{x^9}{(4x^2+1)^6} dx$

$$= \int \frac{x^9 dx}{x^{12}\left(4+\frac{1}{x^2}\right)^6} = \int \frac{dx}{x^3\left(4+\frac{1}{x^2}\right)^6}$$

Put $4 + \frac{1}{x^2} = t \Rightarrow \frac{-2}{x^3} dx = dt$

$$I = -\frac{1}{2} \int \frac{dt}{t^6} = \frac{-1}{2} \int t^{-6} dt$$

$$= -\frac{1}{2} \frac{t^{-5}}{-5} + C = \frac{1}{10}\left(4 + \frac{1}{x^2}\right)^{-5} + C$$

4. (c) Since, $\int \frac{dx}{(x+2)(x^2+1)}$

$$= a\log|1+x^2| + b\tan^{-1} x + \frac{1}{5}\log|x+2| + C$$

Suppose

$$I = \int \frac{dx}{(x+2)(x^2+1)}$$

Let, $\frac{1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

$\therefore \quad 1 = A(x^2+1) + (Bx+C)(x+2)$

$\Rightarrow 1 = (A+B)x^2 + (2B+C)x + A + 2C$

So, $A + B = 0, A + 2C = 1, 2B + C = 0$

$\Rightarrow A = \frac{1}{5}, B = -\frac{1}{5}$ and $C = \frac{2}{5}$

Therefore, $\int \frac{dx}{(x+2)(x^2+1)} = \frac{1}{5}\int \frac{1}{x+2} dx + \int \frac{-\frac{1}{5}x + \frac{2}{5}}{x^2+1} dx$

$$= \frac{1}{5}\int \frac{1}{x+2} dx - \frac{1}{5}\int \frac{x}{1+x^2} dx + \frac{1}{5}\int \frac{2}{1+x^2} dx$$

$$= \frac{1}{5}\log|x+2| - \frac{1}{10}\log|1+x^2| + \frac{2}{5}\tan^{-1} x + C$$

Hence, $a = \frac{-1}{10}, b = \frac{2}{5}$

5. (d) Suppose, $I = \int \frac{x^3}{x+1} dx$

$$= \int \left[(x^2 - x + 1) - \frac{1}{(x+1)}\right] dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \log|x+1| + C$$

6. (d) Suppose, $I = \int \frac{x+\sin x}{1+\cos x} dx$

$$I = \int \frac{x}{1+\cos x} dx + \int \frac{\sin nx}{1+\cos x} dx$$

$$= \int \frac{x}{2\cos^2(x/2)} dx + \int \frac{2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)} dx$$

$$= \frac{1}{2}\int x\sec^2(x/2) dx + \int \tan(x/2) dx$$

$$= \frac{1}{2}\left[2x\tan(x/2) - \int 2\tan\frac{x}{2} dx\right] + \int \tan\frac{x}{2} dx$$

$$= x \cdot \tan\frac{x}{2} + C$$

7. (d) Suppose, $I = \int \frac{x^3}{\sqrt{1+x^2}} dx$

$$= a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$$

Now $I = \int \frac{x^3}{\sqrt{1+x^2}} dx = \int \frac{x^2 \cdot x}{\sqrt{1+x^2}} dx$

Let $1 + x^2 = t^2$

So $2x dx = 2t dt$

Therefore,

$$I = \int \frac{t(t^2-1)}{t} dt = \frac{t^3}{3} - t + C$$

$$= \frac{1}{3}(1+x^2)^{3/2} - \sqrt{1+x^2} + C$$

Hence, $a = \frac{1}{3}$ and $b = -1$

8. (a) Suppose,

$$I = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x} = \int_{-\pi/4}^{\pi/4} \frac{dx}{2\cos^2 x}$$

$$= \frac{1}{2}\int_{-\pi/4}^{\pi/4} \sec^2 x dx = \int_0^{\pi/4} \sec^2 x dx$$

$$= [\tan x]_0^{\pi/4} = 1$$

$$\left[\int_{-a}^{a} f(x) dx = 2\int_0^a f(x) dx \text{ if } f(-x) = f(x)\right]$$

9. (d) Suppose, $I = \int_0^{\pi/2} \sqrt{1 - \sin 2x} dx$

$$= \int_0^{\pi/4} \sqrt{(\cos x - \sin x)^2} dx + \int_{\pi/4}^{\pi/2} \sqrt{(\sin x - \cos x)^2} dx$$

$$\left[\begin{array}{l}\because \cos\ x \geq \sin\ x \text{ for } x \in \left[0 \cdot \frac{\pi}{4}\right] \\ \text{and } \sin\ x \geq \cos\ x \text{ for } x \in \left[\frac{\pi}{4} \cdot \frac{\pi}{2}\right]\end{array}\right]$$

$$= [\sin\ x + \cos\ x]_0^{\pi/4} + [-\cos\ x - \sin\ x]_{\pi/4}^{\pi/2}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - 0 + 1 + \left(-0 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= 2(\sqrt{2} - 1)$$

10. (b) Suppose, $I = \int_0^{\pi/2} \cos\ x e^{\sin\ x} dx$

Let $\sin\ x = t \Rightarrow \cos\ x dx = dt$

$$x \to 0 \Rightarrow t \to 0$$

and $x \to \frac{\pi}{2} \Rightarrow t \to 1$

So $\quad I = \int_0^1 e^t dt = [e^t]_0^1 = e - 1$

11. (a) Suppose,

$$I = \int \frac{x+3}{(x+4)^2} e^x dx = \int \frac{(x+4)1}{(x+4)^2} e^x dx$$

$$= \int \frac{e^x}{(x+4)} - \int \frac{e^x}{(x+4)^2} dx$$

$$= \int e^x \left(\frac{1}{(x+4)} - \frac{1}{(x+4)^2}\right) dx$$

$$= e^x \frac{1}{(x+4)} + C\left[\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C\right]$$

12. (d) $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3} dx = \int \frac{\frac{2}{x^3}+\frac{5}{x^6}dx}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}$

Dividing by x^{15} in numerator and denominator

Substitute $1 + \frac{1}{x^2} + \frac{1}{x^5} = t$

$$\Rightarrow \left(\frac{-2}{x^3} - \frac{5}{x^6}\right) dx = dt \Rightarrow \left(\frac{2}{x^3} + \frac{5}{x^6}\right) dx = -dt$$

This gives,

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6} dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} = \int \frac{-dt}{t^3} = \frac{1}{2t^2} + C$$

$$= \frac{1}{2\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^2} + C = \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$$

13. (c) $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x} I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1-\cos x}$

Adding (i) and (ii)

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2}{\sin^2\ x} dx \Rightarrow I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{cosec}^2\ x dx$$

$$I = -(\cot\ x)_{\pi/4}^{3\pi/4} = -\left[\cot \frac{3\pi}{4} - \cot \frac{\pi}{4}\right] = 2$$

14. (c) $\mathrm{I_n} = \int \tan^{\mathrm{n}} x dx, \mathrm{n} > 1$ Let $\mathrm{I} = \mathrm{I}_4 + \mathrm{I}_6 = \int (\tan^4 x + \tan^6 x)dx = \int \tan^4 x\sec^2 xdx$ Let $\tan x = t \Rightarrow \sec^2 xdx = dt$

$\therefore I = \int \mathrm{t}^4\mathrm{dt} = \frac{\mathrm{t}^5}{5} + \mathrm{C} = \frac{1}{5}\tan^5 \mathrm{x} + \mathrm{C}$

$\Rightarrow$ On comparing, we have $\mathrm{a} = \frac{1}{5}, \mathrm{b} = 0$

15. (a) Let $I = \int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$

$$= \int \frac{\sin^2 x\cos^2 x}{[(\sin^2 x + \cos^2 x)(\sin^3 x + \cos^3 x)]^2}dx$$

$$= \int \frac{\sin^2 x\cos^2 x}{(\sin^3 x + \cos^3 x)^2}dx = \int \frac{\tan^2 x \cdot \sec^2 x}{(1 + \tan^3 x)^2}dx$$

Now, put $(1 + \tan^3 \mathrm{x}) = \mathrm{t}$

$\Rightarrow 3\tan^2 x\sec^2 xdx = dt$

$\therefore \quad \mathrm{I} = \frac{1}{3}\int \frac{\mathrm{dt}}{\mathrm{t}^2} = -\frac{1}{3\mathrm{t}} + \mathrm{C} = \frac{-1}{3(1 + \tan^3 \mathrm{x})} + \mathrm{C}$

16. (c) Let, $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}dx$

Using, $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$, we get :

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^{-x}}dx \qquad (ii)$$

Adding (i) and (ii), we get;

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 xdx \Rightarrow 2I = 2 \cdot \int_0^{\pi/2} \sin^2 xdx$$

$$\Rightarrow \quad 2I = 2 \times \frac{\pi}{4} \Rightarrow I = \frac{\pi}{4}$$

17. (b) $\mathrm{I} = \int_0^\pi |\cos x|^3 dx$

$= 2\int_0^{\pi/2} \cos^3 xdx$

$= \frac{2}{4}\int_0^{\pi/2} (3\cos x + \cos 3x)dx$

$[\because \cos 3\theta = 4\cos^3 \theta - 3\cos \theta]$

$= \frac{1}{2}\left[3\sin x + \frac{\sin 3x}{3}\right]_0^{\pi/2} = \frac{1}{2}\left(3 - \frac{1}{3}\right) = \frac{4}{3}$

18. (c, d) Consider the given integral

$$I = \int x\sqrt{\frac{2\sin (x^2 - 1) - 2\sin (x^2 - 1)\cos (x^2 - 1)}{2\sin (x^2 - 1) + 2\sin (x^2 - 1)\cos (x^2 - 1)}}dx$$

$(\therefore \sin 2\theta = 2\sin \theta\cos \theta)$

$\Rightarrow I = \int x\sqrt{\frac{1 - \cos (x^2 - 1)}{1 + \cos (x^2 - 1)}}dx$

$\Rightarrow I = \int x\left|\tan \left(\frac{x^2-1}{2}\right)\right| dx,$

Now let $\frac{x^2-1}{2} = t \Rightarrow \frac{2x}{2}dx = dt$

$\therefore I=\int|\tan(t)|dt=\ln|\sec t|+C$

or $\quad I=\ln\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c=\frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

19. (a) $I=\int \sec^{\frac{2}{3}} x\cdot \operatorname{cosec}^{\frac{4}{3}} dx$

$I=\int \frac{\sec^2 xdx}{4}$

$\tan^{\overline{3}} x$

Put $\tan x=z$

$\Rightarrow \sec^2 xdx=dz$

$\Rightarrow I=\int z^{\frac{4}{3}}\cdot dz=\frac{Z^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+C\Rightarrow I=-3(\tan x)^{\frac{-1}{3}}+C$

20. (b) Let $I=\int_0^{\pi/2}\frac{\sin^3 xdx}{\sin x+\cos x}$

Use the property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$\Rightarrow I=\int_0^{\pi/2}\frac{\cos^3 xdx}{\sin x+\cos x}$

Adding equations (i) and (ii), we get

$$2I=\int_0^{\pi/2}\left(1-\frac{1}{2}\sin(2x)\right)dx$$

$$\Rightarrow=\frac{1}{2}\left[x+\frac{1}{4}\cos 2x\right]_0^{\pi/2}\Rightarrow I=\frac{\pi-1}{4}$$

21. (d) Let $I=\int\frac{\cos xdx}{\sin^3 x(1+\sin^6 x)^{2/3}}$

If $\sin x=t=f(x)(1+\sin^6 x)^{1/\lambda}+\mathrm{c}$

then, $\cos xdx=dt$

$$I=\int\frac{dt}{t^3(1+t^6)^{\frac{2}{3}}}=\int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Put $1+\frac{1}{t^6}=r^3\Rightarrow\frac{dt}{t^7}=\frac{-1}{2}r^2dr$

$$-\frac{1}{2}\int\frac{r^2dr}{r^2}=-\frac{1}{2}r+c$$

$=-\frac{1}{2}\left(\frac{\sin^6 x+1}{\sin^6 x}\right)^{\frac{1}{3}}+c$

$=-\frac{1}{2\sin^2 x}(1+\sin^6 x)^{\frac{1}{3}}+c$

$f(x)=-\frac{1}{2}\operatorname{cosec}^2 x$ and $\lambda=3$

$\therefore \lambda f\left(\frac{\pi}{3}\right)=-2$

[from eqn. (i)]

22. (3) $I=\int\frac{\frac{\sin x}{\cos^3 x}}{1+\tan^3 x}dx$

$$= \int \frac{\tan x \cdot \sec^2 x}{(\tan x + 1)(1 + \tan^2 x - \tan x)} dx$$

Let $\tan x = t \Rightarrow \sec^3 x \cdot dx = dt$

$$= \int \frac{t}{(t+1)(t^2 - t + 1)} dt$$

Let $\frac{t}{(t+1)(t^2-t+1)} = \frac{A}{t+1} + \frac{B(2t-1)}{t^2-t+1} + \frac{C}{t^2-t+1}$

$\Rightarrow \mathrm{A}(\mathrm{t}^2 - \mathrm{t} + 1) + \mathrm{B}(2\mathrm{t} - 1)(\mathrm{t} + 1) + \mathrm{C}(\mathrm{t} + 1) = \mathrm{t}$

$\Rightarrow \mathrm{t}^2(\mathrm{A} + 2\mathrm{B}) + \mathrm{t}(-\mathrm{A} + \mathrm{B} + \mathrm{C}) + \mathrm{A} - \mathrm{B} + \mathrm{C} = \mathrm{t}$

$$\therefore A + 2B = 0 \qquad (i)$$

$$-A + B + C = 1 \qquad (ii)$$

$$A - B + C = 0 \qquad (iii)$$

$$\Rightarrow \mathrm{C} = \frac{1}{2} \Rightarrow \mathrm{A} - \mathrm{B} = -\frac{1}{2}$$

Solving equations (i) and (iv), we get

$$\mathrm{B} = \frac{1}{6}, \mathrm{A} = -\frac{1}{3}$$

$$\therefore I = -\frac{1}{3}\int \frac{dt}{1+t} + \frac{1}{6}\int \frac{2\mathrm{t} - 1}{\mathrm{t}^2 - \mathrm{t} + 1} \mathrm{dt} + \frac{1}{2}\int \frac{\mathrm{dt}}{\mathrm{t}^2 - \mathrm{t} + 1}$$

$$= -\frac{1}{3}\log_e |(1 + \tan x)| + \frac{1}{6}\log_e |\tan^2 x - \tan x + 1|$$

$$+ \frac{1}{2} \cdot \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\left(\tan x - \frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right)$$

$$= -\frac{1}{3}\log_e |(1 + \tan x)| + \frac{1}{6}\log_e |\tan^2 x - \tan x + 1|$$

$$+ \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$$

$$\alpha = -\frac{1}{3}, \beta = \frac{1}{6}, \gamma = \frac{1}{\sqrt{3}}$$

So, $18(\alpha + \beta + \gamma^2) = 18\left(-\frac{1}{3} + \frac{1}{6} + \frac{1}{3}\right) = 3$

23.

(b) $\pi^2\left[\int_0^1 \sin \frac{\pi x}{2} dx + \int_1^2 \sin \frac{\pi x}{2}(x-1)dx\right]$

$$= \pi^2\left[\left(-\frac{2}{\pi}\left(\cos \frac{\pi \mathrm{x}}{2}\right)\right)_0^1 + \left((\mathrm{x}-1)\left(-\frac{2}{\pi}\cos \frac{\pi \mathrm{x}}{2}\right)\right)_1^2 - \int_1^2 -\frac{2}{\pi}\cos \frac{\pi \mathrm{x}}{2} \mathrm{dx}\right]$$

$$= \pi^2\left[0 + \frac{2}{\pi} + \frac{2}{\pi} + \frac{2}{\pi} \cdot \frac{2}{\pi}\left(\sin \frac{\pi x}{2}\right)_1^2\right]$$

$$= 4\pi - 4 = 4(\pi - 1)$$

24. (c) Given $a = \lim_{\mathrm{n}\to\infty} \sum_{\mathrm{k}=1}^{\mathrm{n}} \frac{2\mathrm{n}}{\mathrm{k}^2 + \mathrm{n}^2}$

$$a = \int_0^1 \frac{2}{x^2 + 1} dx$$

$$\mathrm{a} = 2\tan^{-1}|_0^1 = 2(\tan^{-1}(1) - \tan^{-1}(0))$$

$a = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

Take $f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}} = \left|\tan \frac{x}{2}\right|$

Jiff. w.r.t. x

$$f'(x) = \frac{1}{2}\sec^2 \left.\frac{x}{2}\right|_{x=\frac{\pi}{4}} = \frac{1}{2}\sec^2 \left(\frac{\pi}{8}\right) = \Bigg| \quad \frac{\sqrt{2}}{\sqrt{2}+1}$$

$f'\left(\frac{\pi}{4}\right) = \sqrt{2} f\left(\frac{\pi}{4}\right)$.

25. (24) Given expression is

$$n(2n+1)\int_0^1 (1-x^n)^{2n}dx = 1177\int_0^1 (1-x^n)^{2n+1}dx$$

$$\frac{\int_0^1 (1-x^n)^{2n}dx}{\int_0^1 (1-x^n)^{2n+1}dx} = \frac{1177}{n(2n+1)}$$

Let $I_1 = \int_0^1 (1-x^n)^{2n}dx, I_2 = \int_0^1 (1-x^n)^{2n+1}dx$.

$\Rightarrow \quad I_2 = (1-x^n)^{2n+1}$;

$$x\Big|_0^1 - \int_0^1 (2x+1)\left(1-x^n\right)^{2n}\left(-nx^{n-1}\right)x\,dx$$

$$I_2 = -n(2n+1)\{I_2 - I_1\}$$

$$(2n^2+n+1)I_2 = n(2n+1)I_1$$

$\frac{2n^2+n+1}{n(2n+1)} = \frac{1177}{n(2n+1)}$ from (i) }

$\Rightarrow 2n^2 + n - 1176 = 0$

$2n^2 + 49n - 48n - 1176 = 0$

$(n-24)(2n+49) = 0 = 7 \Rightarrow n = 24$.

26. (c) Since, $I(x) = \int \underset{II}{e^{\sin^2 x}\cdot\sin^2 x} \cdot \underset{I}{\cos x}\, dx - \int e^{\sin^2 x} \cdot \sin x dx = \cos x \int e^{\sin^2 x}\sin 2x dx -$

$$\int \left((-\sin x)\int e^{\sin^2 x}\sin 2x dx\right)dx - \int e^{\sin^2 x}\sin x dx$$

Let $\sin^2 x = t \Rightarrow \sin 2x dx = dt$

$= \cos x \int e^t dt + \int \left(\sin x \int e^t dt\right)dx - \int \sin x e^{\sin^2 x}dx$

$= \cos x e^t + \int e^{\sin^2 x}\sin x dx - \int e^{\sin^2 x}\sin x dx$

Since, $I(0) = 1 \quad \Rightarrow \mathbf{C} = 0$

$\therefore I\left(\frac{\pi}{3}\right) = e^{\frac{3}{4}} \cdot \cos \frac{\pi}{3} = \frac{1}{2}e^{\frac{3}{4}}$

27. (b) Let $I = \int_0^\infty \frac{6}{(e^x+1)(e^x+2)(e^x+3)}dx$

$$= 6\int_0^{\infty}\left(\frac{\frac{1}{2}}{e^x+1}+\frac{-1}{e^x+2}+\frac{\frac{1}{2}}{e^x+3}\right)dx$$
$$= 3\int_0^{\infty}\frac{e^{-x}}{1+e^{-x}}dx - 6\int_0^{\infty}\frac{e^{-x}dx}{1+2e^{-x}} + 3\int_0^{\infty}\frac{e^{-x}}{1+3e^{-x}}dx$$
$$= 3[-\ln(1+e^{-x})]_0^{\infty} + 3[\ln(1+2e^{-x})]_0^{\infty}$$
$$-[\ln(1+3e^{-x})]_0^{\infty}$$
$$= 3\ln 2 - 3\ln 3 + \ln 4 = 3\ln\frac{2}{3} + \ln 4 = \ln\frac{32}{27}$$

EXERCISE - 3

1. (a) Let $x+3 = \mathrm{A}d\,\frac{(3-4x-x^2)}{dx} + \mathrm{B}$

$= \mathrm{A}(-4-2x) + \mathrm{B}$

On comparing we get $A = -\frac{1}{2}$ and $B = 1$

$$\therefore \mathrm{I} = \int -\frac{1}{2}(-4-2x)\sqrt{3-4x-x^2}dx + \int\sqrt{3-4x-x^2}dx$$
$$= -\frac{1}{2}\times\frac{2}{3}(3-4x-x^2)^{3/2} + \int\sqrt{(\sqrt{7})^2-(x+2)^2}dx$$
$$= -\frac{1}{3}(3-4x-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} +$$

2. (b) We have

$$\frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + C$$

$$f(x) = \lim_{n\to\infty}\frac{1}{1-x}\left[(1-x)(1+x)(1+x^2)\ldots\left(1+x^{2^n}\right)\right]$$
$$= \lim_{n\to\infty}\frac{1}{1-x}\left[(1-x^2)(1+x^2)(1+x^4)\ldots\left(1+x^{2^n}\right)\right]$$
$$= \lim_{n\to\infty}\frac{1}{1-x}\left[(1-x^4)(1+x^4)\ldots\left(1+x^{2^n}\right)\right]$$
$$= \lim_{n\to\infty}\left(\frac{1-x^{2^{n+1}}}{1-x}\right) = \frac{1}{1-x}$$

Therefore $f(x) = \frac{1}{1-x}$

Let $I = \int\frac{f(x)}{1-x}\log_e x\,dx = \int\frac{\log_e x}{(1-x)^2}dx$

$$= \left(\frac{1}{1-x}\right)\log_e x - \int \frac{1}{1-x}\cdot\frac{1}{x}dx$$
$$= \frac{\log_e x}{1-x} - \int\left(\frac{1}{1-x}+\frac{1}{x}\right)dx$$
$$= \frac{\log_e x}{1-x} - \log_e\left(\frac{x}{1-x}\right) + c$$

3. (a) We have,

$$\int \frac{x^2-1}{x\sqrt{(x^2+\alpha x+1)(x^2+\beta x+1)}}dx$$

$$= \int \frac{1-\frac{1}{x^2}}{\sqrt{\left(x+\frac{1}{x}+\alpha\right)\left(x+\frac{1}{x}+\beta\right)}}dx$$

$$= \int \frac{d\left(x+\frac{1}{x}\right)}{\sqrt{\left(x+\frac{1}{x}\right)^2+(\alpha+\beta)\left(x+\frac{1}{x}\right)+\alpha\beta}}$$

$= \int \frac{1}{\sqrt{t^2+(\alpha+\beta)t+\alpha\beta}}dt$, where $t = x+\frac{1}{x}$

$$= \int \frac{1}{\sqrt{\left(t+\frac{\alpha+\beta}{2}\right)^2-\left(\frac{\alpha-\beta}{2}\right)^2}}dt$$

$$= \log\left|\left(t+\frac{\alpha+\beta}{2}\right)+\sqrt{\left(t+\frac{\alpha+\beta}{2}\right)^2-\left(\frac{\alpha-\beta}{2}\right)^2}\right|+C_1$$

$$= \log\left|t+\frac{\alpha+\beta}{2}+\sqrt{t^2+(\alpha+\beta)t+\alpha\beta}\right|+C_1$$

$$= \log\{\sqrt{t+\alpha}+\sqrt{t+\beta}\}^2+C_1-\log\ 2$$

$$= 2\log\{\sqrt{t+\alpha}+\sqrt{t+\beta}\}+C_1-\log\ 2$$

$$= 2\log\left\{\frac{\sqrt{x^2+\alpha x+1}+\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\}+C$$

4. (a) Let $x^2 = t \Rightarrow 2xdx = \mathrm{d}t$

$$\frac{1}{2}\int \frac{dt}{t^4+1} = \frac{1}{4}\int \frac{(t^2+1)-(t^2-1)}{t^4+1}dt$$

$$= \frac{1}{4}\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt - \frac{1}{2}\int \frac{x^5-x}{x^8+1}dx$$

$$= \frac{1}{4}\int \frac{1+\frac{1}{t^2}dt}{\left(t-\frac{1}{t}\right)^2+2} - \frac{1}{2}\int \frac{x^5-x}{x^8+1}dx$$

$$= \frac{1}{4\sqrt{2}}\tan^{-1}\left(\frac{x^4-1}{\sqrt{2}x^2}\right) - \frac{1}{2}\int\left(\frac{x^5-x}{x^8+1}\right)dx + C$$

5. (b) We have $\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}}$

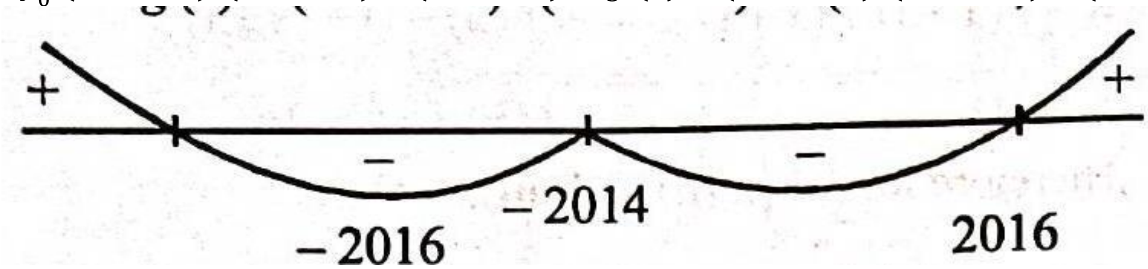
$$= \int \frac{dx}{x^2\cdot x^{n-1}\left(1+\frac{1}{x^n}\right)^{(n-1)/n}} = \int \frac{dx}{x^{n+1}(1+x^{-n})^{(n-1)/n}}$$

Put $1+x^{-n} = t$

$\therefore -nx^{-n-1}dx = dt$ or $\frac{dx}{x^{n+1}} = -\frac{dt}{n}$ then

$$\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}} = -\frac{1}{n}\int \frac{dt}{t^{(n-1)/n}} = -\frac{1}{n}\int t^{\frac{1}{n}-1}dt$$
$$= -\frac{1}{n}\frac{t^{1/n-1+1}}{1/n-1+1} + c = -t^{1/n} + c = -(1+x^{-n})^{1/n} + c$$

6. (b) $g(x) = \int_0^x (t+2016)^7(t+2014)^{110}(t-2016)^5 \mathrm{dt} g'(x) = (x+2016)^7(x+2014)^{110}(x-2016)^5$



at $x = -2016$ is local max.
$x = 2016$ is local min.

$$I = \int_{-2016}^{2016} \frac{dx}{1+x^{2n+1}+\sqrt{1+x^{4n+2}}}$$
$$I = \int_{-2016}^{2016} \frac{1+x^{2n+1}\sqrt{1+x^{4n+2}}}{2x^{2n+1}}\mathrm{dx}$$
$$I = \int_{-2016}^{2016} \frac{1-\sqrt{1+x^{4n+2}}}{2x^{2n+1}}\mathrm{dx} + \frac{1}{2}\int_{-2016}^{2016} 1dx$$
$$= 0 + \frac{1}{2}[2016+2016] = 2016$$

7.

$$I = \int_\alpha^\beta \frac{e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\alpha}\right)}}{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\alpha}\right) e_e \left(\frac{a(x-\alpha)(x-\beta)}{x-\beta}\right)}dx$$

(c) $$= \int_\alpha^\beta \frac{e^{f(a(\alpha+\beta-x-\beta))}}{e^{f(a(\alpha+\beta-x-\beta))} + e^{f(a(\alpha+\beta-x-\alpha))}dx}$$

$$2I = \int_\alpha^\beta dx \Rightarrow I = \frac{|\alpha-\beta|}{2} = \frac{\sqrt{b^2-4ac}}{2|a|}$$

8. (b) Let $p(x) = (g(x))^2 - h(x)$
$p'(x) = f(x)(2g(x) - (f(x))^2)$
$\mathrm{q}'(\mathrm{x}) = 2\mathrm{f}(\mathrm{x})(1-\mathrm{f}(\mathrm{x})) \geq 0 \forall \mathrm{x}$
$\Rightarrow \mathrm{q}(\mathrm{x}) \geq 0 \forall \mathrm{x}$
$\Rightarrow \mathrm{p}'(\mathrm{x}) \geq 0 \forall \mathrm{x}$
$\Rightarrow \mathrm{p}(\mathrm{x}) \geq 0 \forall \mathrm{x} \in [0,1]$

9. (a) Let $q = p+d, r = p+2d, s = p+3d$

$$\therefore f(x) = \begin{vmatrix} p+\sin x & p+d+\sin x & -2d+\sin x \\ p+d+\sin x & p+2d+\sin x & -1+\sin x \\ p+2d+\sin x & p+3d+\sin x & 2d+\sin x \end{vmatrix}$$

Applying $R \rightarrow R_1 + R_3 - 2R_2$, we get

$$f(x)=\begin{vmatrix} 0 & 0 & 2 \\ p+d+\sin x & p+2d+\sin x & -1+\sin x \\ p+2d+\sin x & p+3d+\sin x & 2d+\sin x \end{vmatrix}$$
$$= 2[(p+d+\sin x)(p+3d+\sin x)$$
$$-(p+2d+\sin x)^2 = -2d^2$$

Given $\int_0^2 f(x)dx = -4$

$\Rightarrow \quad \int_0^2 (-2d^2)dx = -4 \Rightarrow d^2 = 1 \Rightarrow d = \pm 1$

10. (b) $g(x) = \int_0^x f(t)dt$

$\Rightarrow g(2) = \int_0^2 f(t)dt = \int_0^1 f(t)dt + \int_1^2 f(t)dt$

Now, $\frac{1}{2} \leq f(t) \leq 1$ for $t \in [0,1]$

We get $\int_0^1 \frac{1}{2} dt \leq \int_0^1 f(t)dt \leq \int_0^1 1dt$

(applying line integral on inequality)

$\Rightarrow \quad \frac{1}{2} \leq \int_0^1 f(t)dt \leq 1$

Again, $0 \leq f(t) \leq \frac{1}{2}$ for $t \in [1,2]$

We get $\int_1^2 0dt \leq \int_1^2 f(t)dt \leq \int_1^2 \frac{1}{2} dt$

(applying line integral on inequality)

$\Rightarrow \quad 0 \leq \int_1^2 f(t)dt \leq \frac{1}{2}$

From (i) and (ii), we get $\frac{1}{2} \leq \int_0^1 f(t)dt + \int_1^2 f(t)dt \leq \frac{3}{2}$

or $\quad \frac{1}{2} \leq g(2) \leq \frac{3}{2}$

$\Rightarrow 0 \leq g(2) \leq 2$ is the most appropriate solution.

11. (a) $\lim_{x\to 1} \frac{\int_4^{f(x)} 2tdt}{x-1} = \lim_{x\to 1} \frac{(t^2)|_4^{f(x)}}{x-1}$

$= \lim_{x\to 1} \frac{(f(x))^2 - 4^2}{x-1}$

$= \lim_{x\to 1} \frac{2f(x)\cdot f'(x)}{1}$

(L-Hospital rule)

$= 2f(1)\cdot f'(1) = 2\times 4\times 2 = 16$

12. (a) We have $\lim_{x\to 0}\left(1+x+\frac{f(x)}{x}\right)^{1/x} = e^3$

$\Rightarrow \lim_{x\to 0}\left(1+x\left(1+\frac{f(x)}{x^2}\right)\right)^{1/x} = e^3$

$\Rightarrow \lim_{x\to 0}\left[\left(1+\frac{x^2+f(x)}{x}\right)^{\frac{x}{x^2+f(x)}}\right]^{\frac{x^2+f(x)1}{x}} = e^3$

$\Rightarrow \frac{x^2+f(x)}{x^2} = 3 \Rightarrow f(x) = 2x^2$

Therefore,

$\int f(x)\log_e\ x dx = 2\int x^2\log_e\ x dx$

$= 2\left[\frac{x^3}{3}\log_e\ x - \int \frac{x^3}{3}\cdot\frac{1}{x}dx\right]$ (By Parts)

$= \frac{2}{3}x^3\log_e\ x - \frac{2}{9}x^3 + c = \frac{2}{3}x^3\left(\log_e\ x - \frac{1}{3}\right) + c$

13. (a) First, we determine $f(x)$.

$f(0) = f(-0) = -f(0) \Rightarrow f(0) = 0$

and $0 = f(0) = f(1 + (-1)) = f(-1) + 1 \Rightarrow f(-1) = -1$

Therefore $f(0) = 0$ and $f(-1) = -1$. Let $x \neq 0$ and -1 . From (ii), we have

$$f\left(\frac{1}{x} + 1\right) = f\left(\frac{1}{x}\right) + 1 = \frac{f(x)}{x^2} + 1 \quad \text{[By (iii)]} \qquad (i)$$

Therefore $f\left(\frac{1+x}{x}\right) = \frac{f(x)}{x^2} + 1$

Again

$$f\left(\frac{x}{1+x}\right) = f\left(\frac{1}{\left(\frac{1+x}{x}\right)}\right) = \frac{f\left(\frac{1+x}{x}\right)}{\left(\frac{1+x}{x}\right)^2}$$

$$= \left(\frac{x}{1+x}\right)^2\left(\frac{f(x)}{x^2} + 1\right)$$

[By Eq. (i)]

Therefore

$$(1+x)^2 f\left(\frac{x}{1+x}\right) = f(x) + x^2 \qquad (ii)$$

Also

$(1+x)^2 f\left(\frac{x}{1+x}\right) = (1+x)^2 f\left(1 - \frac{1}{x+1}\right)$

$= (1+x)^2 f\left(\frac{-1}{x+1} + 1\right) = (1+x)^2\left[-f\left(\frac{1}{x+1}\right) + 1\right]$

$[\because f(-x) = -f(x)]$

$$= (1+x)^2\left[\frac{-f(x+1)}{(x+1)^2} + 1\right] \quad \left(\because f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2}\right)$$

$$= -f(x+1) + (x+1)^2 = -(f(x) + 1) + (x+1)^2$$

$$= -f(x) + x^2 + 2x$$

Therefore from eqs. (ii) and (iii), we have

$$f(x) + x^2 = -f(x) + x^2 + 2x$$

$$\Rightarrow 2f(x) = 2x \Rightarrow f(x) = x$$

Thus $f(0) = 0, f(-1) = -1$ and $f(x) = x \forall x \neq 0, -1$.

So $f(x) = x \forall x \in \mathbf{R}$.

Hence $\int e^x f(x)dx = \int e^x x dx$

$$= xe^x - \int e^x dx$$

$$= xe^x - e^x + c = e^x(x-1) + c$$

14. (a) $\int \sec^{2010}\ x\operatorname{cosec}^2\ x dx - \int 2010\sec^{2010}\ x dx = \int \sec^{2010}\ x(-\cot\ x) - \int 2010\sec^{2010}\ x\cdot\tan\ x$

$$\cdot(-\cot\ x) - \int 2010\sec^{2010}\ x dx$$

$$= -\frac{\cot x}{(\cos x)^{2010}} + 2010\int \sec^{2010} xdx$$
$$-2010\int \sec^{2010} xdx + C$$
$$= \frac{-\cot x}{(\cos x)^{2010}} + C \therefore \frac{f(x)}{g(x)} = \frac{1}{\sin x} = \{x\}$$
No solution.

15. (d) We have,
$$\int \frac{1}{x}[\log_{ex} e \cdot \log_{e^2x} \cdot \log_{e^3x} e]dx$$
$$= \int \frac{1}{x\log_e \text{ex} \log_e e^2x\log_e e^3x}dx$$
$$= \int \frac{1}{x(\log_e e + \log_e x)(\log_e e^2 + \log_e x)}dx$$
$$(\log_e e^3 + \log_e x)$$
$$= \int \frac{d(\log_e x)}{(1 + \log_e x)(2 + \log_e x)(3 + \log_e x)}$$
$= \int \frac{1}{(1+t)(2+t)(3+t)}dt$, where $t = \log_e x$
$$= \int \left(\frac{1}{2} \cdot \frac{1}{1+t} - \frac{1}{2+t} + \frac{1}{2} \cdot \frac{1}{3+t}\right)dt$$
$$= \frac{1}{2}\log|1 + \log_e x| - \log|2 + \log_e x| + \frac{1}{2}\log|3 + \log_e x| + C$$
$$= \frac{1}{2}\log\{\log_e ex\} - \log\{\log_e e^2x\} + \frac{1}{2}\log\{\log_e e^3x\} + C$$

EXERCISE - 4

1. (10) Put $1 + x^2 = t^2$
$$\Rightarrow 2xdx = 2tdt$$
$$xdx = tdt$$
Now $\int_1^2 \frac{15(t^2-1)tdt}{\sqrt{t^2+t^3}} = 15\int_1^2 \frac{t(t^2-1)}{t\sqrt{1+t}}dt$

Put $1 + t = u^2$
$$\Rightarrow dt = 2udu$$
$$15\int_{\sqrt{2}}^{\sqrt{3}} \frac{(u^2-1)^2-1}{u} \times 2udu$$
$$30\int_{\sqrt{2}}^{\sqrt{3}} (u^4 - 2u^2)du$$
$$30\left(\frac{u^5}{5} - \frac{2u^3}{3}\right)_{\sqrt{2}}^{\sqrt{3}}$$
$$30\left[\frac{1}{5}\left(\sqrt{3}^5 - \sqrt{2}^5\right) - \frac{2}{3}\left(\sqrt{3}^3 - \sqrt{2}^3\right)\right]$$
$$30\left[\frac{1}{5}(9\sqrt{3} - 4\sqrt{2}) - \frac{2}{3}(3\sqrt{3} - 2\sqrt{2})\right]$$
$$30\left[-\frac{1}{5} \times \sqrt{3} + \frac{8}{15}\sqrt{2}\right]$$
$$-6\sqrt{3} + 16\sqrt{2} = \alpha\sqrt{2} + \beta\sqrt{3}$$

$\Rightarrow \alpha = 16, \beta = -6$

2. (104)

$\therefore \quad \alpha + \beta = 10$

$I = 60\int_0^{\pi/2} \left(\frac{\sin\ 6x - \sin\ 4x}{\sin\ x} + \frac{\sin\ 4x - \sin\ 2x}{\sin\ x} + \frac{\sin\ 2x}{\sin\ x}\right) dx$

$\left(\because \sin\ x - \sin\ y = 2\sin\ \frac{(x+y)}{2} \cdot \cos\ \frac{(x-y)}{2}\right)$

$I = 60\int_0^{\pi/2} (2\cos\ 5x + 2\cos\ 3x + 2\cos\ x) dx$

$I = 60\left(\frac{2}{5}\sin\ 5x + \frac{2}{3}\sin\ 3x + 2\sin\ x\right)\Big|_0^{\pi/2} = 104$

3. (385) Given function is $\mathrm{f(x) = [x] - 10}$

Take, $\int_0^{10} \mathrm{f(x) \cdot dx = -10 - 9 - 8 - \cdots .. -1}$

$= -\frac{10 \cdot 11}{2} = -55$

Take,

$\int_0^{10} (f(x))^2 dx = 10^2 + 9^2 + 8^2 + \cdots + 1^2 = \frac{10 \cdot 11 \cdot 21}{6} = 385$

Take, $\int_0^{10} |f(x)| = 10 + 9 + 8 + \cdots + 1 = \frac{10 \cdot 11}{2} = 55$

$= -55 + 385 + 55 = 385$

4. (12) Let $\frac{\lambda^2 \mathrm{x}}{3} = \ell$

$\Rightarrow \frac{2\lambda \mathrm{x}}{3} \mathrm{d}\lambda = \mathrm{d}\ell \Rightarrow \mathrm{d}\lambda = \frac{3}{2} \cdot \frac{1\sqrt{\mathrm{x}}}{\mathrm{x} \cdot \sqrt{3}\sqrt{\ell}} \mathrm{d}\ell.$

$\Rightarrow \mathrm{d}\lambda = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{\mathrm{x}}} \cdot \frac{\mathrm{dt}}{\ell}$

So, $f(x) = \frac{1}{\sqrt{x}} \int_0^x \frac{f(t)}{\sqrt{t}} dt$

$\Rightarrow \sqrt{x} \cdot f'(x) + \frac{f(x)}{2\sqrt{x}} = \frac{f(x)}{\sqrt{x}}$

$\Rightarrow \sqrt{\mathrm{x}} \cdot \mathrm{f'(x)} = \frac{\mathrm{f(x)}}{2\sqrt{\mathrm{x}}} \Rightarrow \frac{\mathrm{dy}}{\mathrm{y}} = \frac{\mathrm{dx}}{2\mathrm{x}}$

Take integral both sides,

$\Rightarrow \ln\ y = \frac{1}{2}\ln\ x + c \Rightarrow f(x) = \sqrt{x}$

Here, $f(1) = \sqrt{3} \Rightarrow y = \sqrt{3x}$

So, $f(x) = \sqrt{3x}$

Now, $f(\alpha) = 6 \Rightarrow 36 = 3\alpha \Rightarrow \alpha = 12$

5. (8) Given integral is

$f(\mathrm{x}) + \int_0^x (\mathrm{x - t})\mathrm{f(t)dt} = (e^{2x} + e^{-2x})$

$\cos\ 2x + \frac{2x}{a}$

Here $\mathrm{f(0) = 2}$

Differentiate equation (i) w.r.t. x,

$$f'(\mathrm{x})+\int_0^x f'(\mathrm{t})\mathrm{dt}+\mathrm{xf(x)}-\mathrm{xf(x)}=2(\mathrm{e}^{2x}-\mathrm{e}^{-2x})$$

$$\cos\ 2x-2(e^{2x}+e^{2x})\sin\ 2x+\frac{2}{a}$$

$$\Rightarrow f(\mathrm{x})+\mathrm{f(x)}-\mathrm{f(0)}=2(\mathrm{e}^{2x}-\mathrm{e}^{2x})\cos\ 2\mathrm{x}-2(\mathrm{e}^{2x}+\mathrm{e}^{-2x})$$

$$\sin\ 2\mathrm{x}+\frac{2}{a}$$

Replace x by 0 we get :

$\Rightarrow 4=\frac{2}{a}\Rightarrow a=\frac{1}{2}$.

Therefore, $(2a+1)^5\cdot a^2=2^5\cdot\frac{1}{2^2}=2^3=8$

6. (5) Given integral is

$$a_n=\int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{3}+\cdots.+\frac{x^{n-1}}{n}\right)dx$$

$$=\left[x+\frac{x^2}{2^2}+\frac{x^3}{3^2}+\cdots.+\frac{x^n}{n^2}\right]_{-1}^{n}$$

Then, $a_n=\frac{n+1}{1^2}+\frac{n^2-1}{2^2}+\frac{n^3+1}{3^2}+\frac{n^4-1}{4^2}$

$$+\cdots.+\frac{n^n+(-1)^{n+1}}{n^2}$$

Here $a_1=2, a_2=\frac{2+1}{1}+\frac{2^2-1}{2}=3+\frac{3}{2}=\frac{9}{2}$

$$a_3=4+2+\frac{28}{9}=\frac{100}{9}$$

$a_4=5+\frac{15}{4}+\frac{65}{9}+\frac{255}{16}>31.$

$\therefore$ The required set is $\{2,3\}$. $\because \mathrm{a_n}=\in(2,30)$

Hence, sum of elements $=5$.

7.

(2) $\int_0^x f(t)dt=x^2+\int_x^1 t^2f(t)dt$

$$\Rightarrow f(x)=2x-x^2f(x)$$

$$\Rightarrow f(x)=\frac{2x}{1+x^2}$$

$$\Rightarrow f'(x)=\frac{2(1-x^2)}{(1+x^2)^2}$$

Then,

$$f'(1/2)=\frac{2\left(1-\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2}=\frac{3}{2}\times\frac{16}{25}=\frac{24}{25}$$

$$\Rightarrow\frac{24}{5^k}=\frac{24}{25}\quad\Rightarrow\frac{24}{5^k}=\frac{24}{5^2}\quad\Rightarrow k=2$$

8. (1) $I_n+I_{n+2}=\int_0^{\pi/4}\tan^n\ x(1+\tan^2\ x)dx$

$$=\int_0^{\pi/4} \tan^n\ x\sec^2\ xdx=\left[\frac{\tan^{n+1}\ x}{n+1}\right]_0^{\pi/4}$$

$$=\frac{1-0}{n+1}=\frac{1}{n+1}$$

$$\therefore I_n+I_{n+2}=\frac{1}{n+1}\Rightarrow \lim_{n\to\infty} n[I_n+I_{n+2}]$$

$$=\lim_{n\to\infty} n\cdot\frac{1}{n+1}=\lim_{n\to\infty}\frac{n}{n+1}=\lim_{n\to\infty}\frac{n}{n\left(1+\frac{1}{n}\right)}=1$$

9. (0.5) Given f is a function and

$$I_1=\int_{1-k}^{k} xf[x(1-x)]dx$$

$$I_2=\int_{1-k}^{k} f[x(1-x)]dx$$

Now, $I_1=\int_{1-k}^{k} xf[x(1-x)]dx$

$\int_{1-k}^{k}(1-x)f[(1-x)x]dx$

[Using Property $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\Big]$ Adding (i) and (ii)

$2I_1=\int_{1-k}^{k} f[x(1-x)]dx=I_2$

$\therefore \frac{I_1}{I_2}=\frac{1}{2}=0.5$

10. (0) We have, $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[f(x)+f(-x)][g(x)-g(-x)]dx$ Let $F(x)=(f(x)+f(-x))(g(x)-g(-x))$, then

$F(-x)=(f(-x)+f(x))(g(-x)-g(x))$

$=-[f(x)+f(-x)][g(x)-g(-x)]$

$=-F(x)$

$\therefore F(x)$ is an odd function.

Using the property $\int_{-a}^{a} f(x)dx=0$,

if $f(-x)=f(x)$, we get $I=0$

11. (1)

Let, $\int \frac{dx}{x^3(1+x^6)^{\frac{2}{3}}}=\int \frac{dx}{x^7(1+x^{-6})^{\frac{2}{3}}}$

Put $1+x^{-6}=t^3 \Rightarrow -6^{-7}dx=3t^2dt$

$\Rightarrow \frac{dx}{x^7}=\left(-\frac{1}{2}\right)t^2dt$

Now, $I=\int\left(-\frac{1}{2}\right)\frac{t^2dt}{t^2}=-\frac{1}{2}t+C$

$2\sqrt{5}=-\frac{1}{2}(1+x^{-6})^{\frac{1}{3}}+C=-\frac{1}{2}\frac{(1+x^6)^{\frac{1}{3}}}{x^2}+C$

$=-\frac{1}{2x^3}x(1+x^6)^{\frac{1}{3}}+C$

Hence, $f(x)=-\frac{1}{2x^3}\Rightarrow\frac{-C}{2x^3}=\frac{-1}{2x^3}\Rightarrow C=1$

12. (1) $\int_0^1 x\cot^{-1}(1-x^2+x^4)dx = \int_0^1 x\tan^{-1}\left(\frac{1}{1+x^4-x^2}\right)$

$= \int_0^1 x\tan^{-1}\left(\frac{x^2-(x^2-1)}{1+x^2(x^2-1)}\right)dx.$

$= \frac{1}{2}\int_0^1 1\tan^{-1} t^2 dt - \frac{1}{2}\int_{-1}^0 1\tan^{-1} kdk$

Put $x^2 = t \Rightarrow 2xdx = dt$ in the first integral

and $x^2 - 1 = k \Rightarrow 2xdx = dk$ in the second integral.

$$= \frac{1}{2}\int_0^1 1\tan^{-1} tdt - \frac{1}{2}\int_0^1 1\tan^{-1} kdk$$

$$= \frac{1}{2}\int^{-1} t\, t\tan^1 t\Big|_0^1 \frac{t}{1+t^2}dt\Bigg)$$

$$-\frac{1}{2}\left(k\tan^{-1} k|_0^1 - \int_{-1}^0 \frac{k}{1+k^2}dk\right)$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - \left(\frac{1}{2}\ln(1+t^2)\Big|_0^1\right)\right.$$

$$\left.-\frac{1}{2}\left(-\frac{\pi}{4} - \left(\frac{1}{2}\ln(1+k^2)\Big|_{-1}^0\right)\right)\right)$$

$$= \left(\frac{\pi}{8} - \frac{1}{4}\ln 2\right) - \left(\frac{-\pi}{8} - \frac{1}{4}10 - \ln 2\right) = \frac{\pi}{4} - \frac{1}{2}\ln 2$$

$$\Rightarrow \frac{\pi}{4} - \frac{1}{2}\text{In } 2 = \frac{\pi}{4} - \frac{k}{2}\text{In } 2$$

$$\Rightarrow k = 1$$